1. QFT FOR BASIC SISO SYSTEMS: A QUICK REVIEW

When using the Horowitz design method, or QFT, one assumes in general a canonic two degree-of-freedom structure (Figure 1.1). The robust control problem is defined as the problem to find a feedback compensator $G(s)$ and, where appropriate, a prefilter $F(s)$, such that the closed loop specifications are satisfied for all plant cases. In QFT, $G(s)$ is used to reduce the closed loop uncertainty and reject disturbances, and $F(s)$ is used to shape the closed loop transfer functions from reference to output. Instead of solving the control design problem simultaneously for all plant cases, in QFT the problem is transformed to a conventional feedback design problem for one nominal plant only, with frequency dependent constraints, so called Horowitz bounds, in the complex plane on $G(j\omega)$, or equivalently on the nominal open loop. The QFT design then proceeds in six steps:

1. Determine the set of plant transfer functions $\{P_i(s)\}$ for which the control system is to be designed. Assign one arbitrary transfer function, $P_{\text{nom}}(s)$, as the nominal plant. For each of a wisely selected set of frequencies $\{\omega_k\}$ [rad/s], compute the plant templates, or value sets, $\{P_i(j\omega_k)\}$.
2. Determine the closed loop specifications in the time and frequency domains. The specifications given in the time domain are translated to the frequency domain.
3. From the plant templates, and the frequency domain specifications, compute the Horowitz bounds for $\{\omega_k\}$.
4. Display the nominal open loop, $P_{\text{nom}}(j\omega)G(j\omega)$, in a Nichols chart, while designing $G(j\omega)$ by classical loop shaping such that the nominal open loop satisfies the Horowitz bounds at each frequency $\{\omega_k\}$. Using the Nyquist criterion, ascertain that the closed loop is asymptotically stable for all plant cases.
5. Close the loop, $\{G(s)P_i(s)/(1+G(s)P_i(s))\}$, and loop shape $F(s)$, with the aid of a closed loop Bode diagram, such that closed loop transfer function from the reference to the output, $F(s)G(s)P(s)/(1+G(s)P(s))$, falls within its specifications.
6. Simulate the closed loop transfer function for a number of plant cases, and ascertain that the time domain specifications are satisfied.

This chapter covers the design steps in somewhat greater detail. A detailed Qsyn design example is found in Chapter 2. You may want to refer to both these chapters simultaneously.

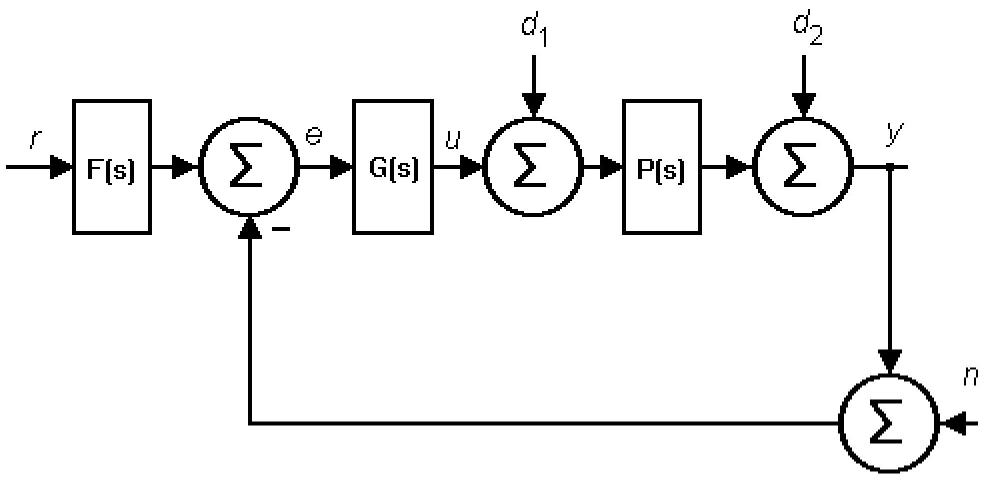

Figure 1.1. Canonic two degree-of-freedom SISO system.

1.1 Two degree-of-freedom system

When using the Horowitz design method, or QFT, in the SISO case, one may assume, without loss of generality, that a *canonical two degree-of-freedom system* is to be designed, see Figure 1.1. In Figure 1.1, $P(s)$ denotes the transfer function for the uncertain plant or controlled object, which may include actuator and sensor dynamics, $G(s)$ is the feedback compensator, and $F(s)$ is the prefilter. The reference signal is called $r(t)$, the loop error $e(t)$, the control signal $u(t)$, the disturbance at the plant input $d_1(t)$, the disturbance at the plant output $d_2(t)$, the system output $y(t)$, and the measurement noise $n(t)$. The Laplace transforms of these signals are denoted $R(s)$, $E(s)$, $U(s)$, $D_1(s)$, $D_2(s)$, $Y(s)$, and $N(s)$, respecively. The disturbances and measurement noise may themselves be filtered signals, and thus any linear feedback system with the reference and output independently measurable may be transformed to the configuration of Figure 1.1.

In those cases when the reference and output are not independently measurable, but only the loop error $e(t)$, like in many tracking problems, we have a *one degree-of-freedom system* in which only $G(s)$ may be designed.

1.2 Plant uncertainty

An *uncertain plant transfer function* is defined as a member of a set of transfer functions

$$P(s) \in \{P_i(s)\} \tag{1.1}$$

where the set may contain finitely or (countably or uncountably) infinitely many plant cases. For each frequency, the set in the complex plane, $\{P_i(j\omega)\}$ is called the *template* or *value set* for ω.

Sometimes it is possible to specialize the definition. A plant having *parametric uncertainty* is defined as

$$P(s) \in \{P(s,q)\}, \quad q \in Q \subset \Re^p \tag{1.2}$$

where q is the vector of uncertain parameters. Another specialized form of plant uncertainty is the *multiplicative unstructured uncertainty*,

$$P(s) = P_i(s)(1+M(s)), \quad |M(s)| \le m(s) \le 1, \quad M(s) \in C, \quad m(s) \in \Re \tag{1.3}$$

where $M(s)$ is an asymptotically stable transfer function, and $P_i(s)$ is a single transfer function, or one of the uncertain plant cases defined in (1.1) or (1.2).

In QFT, *one arbitrary plant* which is *not necessarily* one of the plant cases in (1.1) - (1.3), is selected as the *nominal* plant, $P_{\text{nom}}(s)$. Figure 1.2 illustrates (1.3) with $P_i(s) = P_{\text{nom}}(s)$ in the Nyquist diagram, whereby the frequency function $P_{\text{nom}}(j\omega)$ is surrounded in each point by a circular template with radius = $|P_{\text{nom}}(j\omega)|m(j\omega)$.

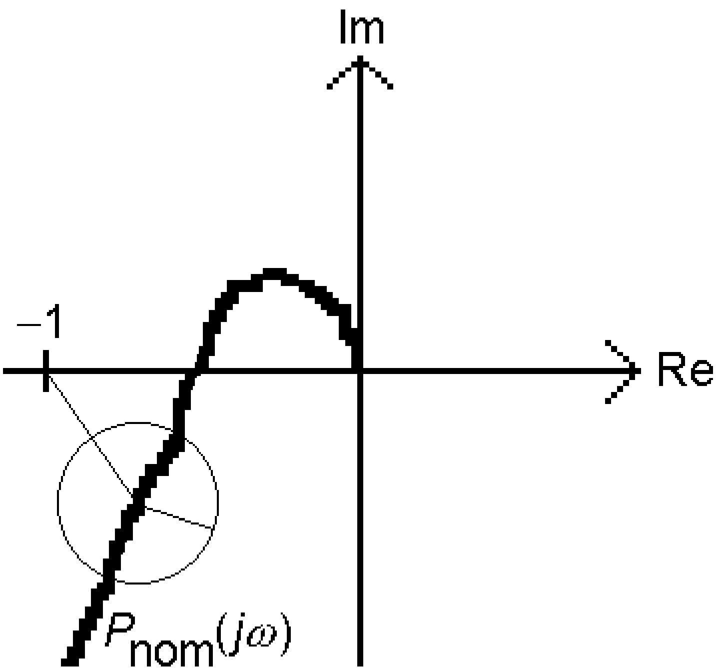

Figure 1.2. The Nyquist plot of an uncertain plant $P(s)$ with unstructured multiplicative uncertainty (1.3) where $P_i(s) = P_{\mathrm{nom}}(s)$. The nominal plant $P_{\mathrm{nom}}(j\omega)$ is marked by a solid line. The radius of the circular uncertainty template has the length $|P_{\mathrm{nom}}(j\omega)|m(j\omega)$.

1.3 Specifications

Specifications may of course be given in many different forms depending on the nature of the problem at hand. Often an envelope for the reference step response is specified, see Figure 1.3 for an example, or an envelope for a disturbance step response, see Figure 1.4. One may specify the measurement noise response envelope, either in the time or frequency domain. Let $R(s)$, $E(s)$, $U(s)$, $D_1(s)$, $D_2(s)$, $Y(s)$, and $N(s)$ be the Laplace transforms of the signals r, e, u, d_1, d_2, y, and n in Figure 1.1, respectively. It is common to assign specifications to the *error transfer function* or *sensitivity function,*

$$S(s) = \frac{E(s)}{R(s)} = \frac{Y(s)}{D_2(s)} = \frac{1}{1+P(s)G(s)} \tag{1.4}$$

in e.g. the form

$$|S(j\omega)| \le x(\omega) \tag{1.5}$$

where $x(\omega) \in \Re$ may be constant, and in any case taking note that at least for some frequencies $x(\omega) > 1$, see e.g. Bode (1943) or Horowitz (1963). There are many other types of specifications, such as minimum phase and gain margins, minimum delay margin, minimum modulus margin, maximum error and stiffness coefficients, etc, etc.

I*n QFT all specifications must be finally formulated in the frequency domain,* and those that are originally given in the time domain must be translated to the frequency domain. Unfortunately there is no one-to-one mapping between specifications in the time and frequency domains. In general one proceeds with the help of the classical dominant pole assumption, i.e. the closed loop, e.g. $F(s)G(s)P(s)/(1+G(s)P(s))$ or $1/(1+G(s)P(s))$, is approximated by a low order dynamic system, e.g. of order 2 or 3, from which the correspondence between the time and frequency domains is found. The procedure is illustrated in the following example.

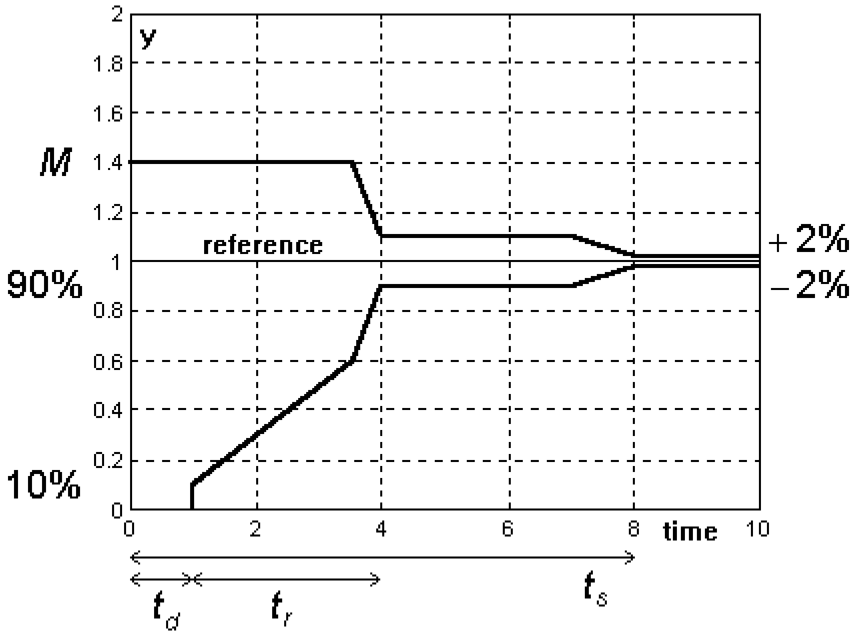

Figure 1.3. Reference step response specification envelope. M is the maximally allowed overshoot (% of step size), t_d is called the delay time, t_r is called the rise time, and t_s is called the settling time.

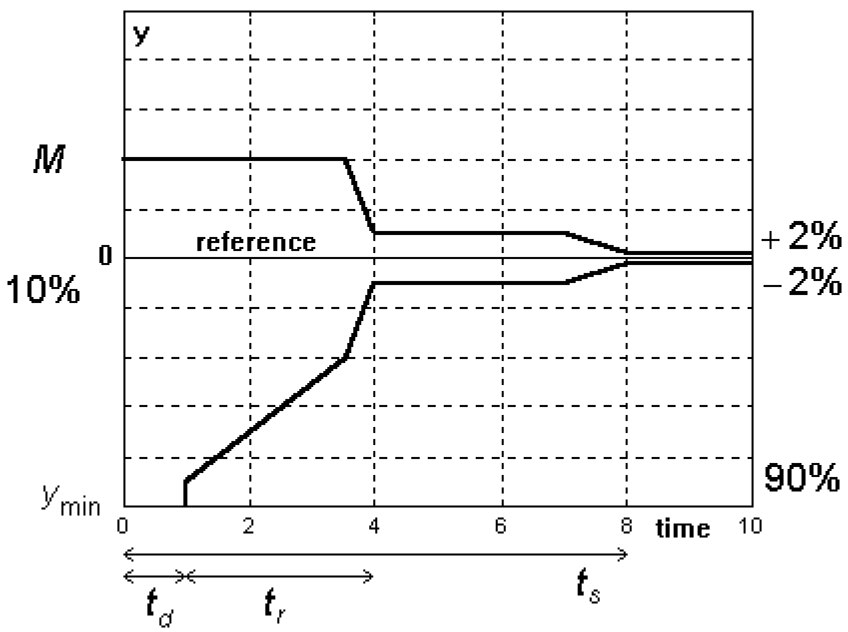

Figure 1.4. Negative disturbance step response specification envelope. $y_{\min}$ is the maximally allowed overshoot (% of disturbance step size), M is the maximally allowed undershoot (% of disturbance step size), t_d is called the delay time, t_r is called the rise time, and t_s is the settling time.

Example 1.1

Consider the reference step response specification in Figure 1.3, and let the upper limit be $u(t)$ and the lower limit $l(t)$. Find the set of all second order systems, $C(s)$, whose step responses satisfy the limits:

$$C(s)=\left\{\frac{\omega_0^2}{s^2+2\varsigma\omega_0 s+\omega_0^2}\right\},\quad (\varsigma,\omega_0)\in Q \tag{1.6}$$

where

$$Q=\left\{(\varsigma,\omega_0)\,\middle|\, l(t)\le L^{-1}\left\{\frac{\omega_0^2}{s^2+2\varsigma\omega_0 s+\omega_0^2}\cdot\frac{1}{s}\right\}\le u(t)\right\} \tag{1.7}$$

and L^{-1} denoting the inverse Laplace transform. The specification translated to the frequency domain would e.g. be

$$a(\omega)\le\left|\frac{F(j\omega)G(j\omega)P(j\omega)}{1+G(j\omega)P(j\omega)}\right|\le b(\omega) \tag{1.8}$$

with

$$a(\omega)=\min_Q\left|C(j\omega)\right|,\quad b(\omega)=\max_Q\left|C(j\omega)\right| \;. \tag{1.9}$$

High frequency roll off

The lower border $a(\omega)$ is in general modified for high frequencies to allow a faster roll-off than -20 dB/dec, since the true closed loop system is of an order (much) larger than two. In Qsyn, reference step response specifications are created and translated to a frequency main specification with the command `rsrs`. The roll-off modification is done with the command `spcupd`. A frequency domain specification generated by `rsrs` that roughly corresponds to the time domain specification in Figure 1.3 is found in Figure 1.5.

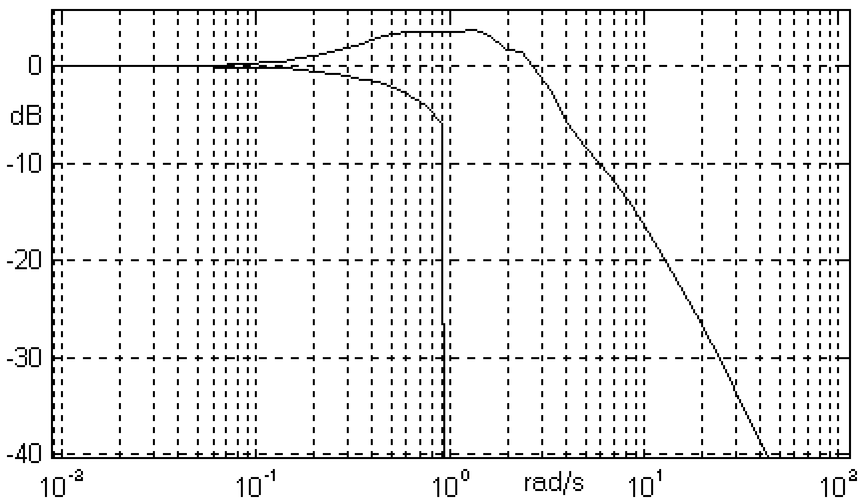

Figure 1.5. Frequency domain specification of type (1.8) generated by the Qsyn command `rsrs` *and displayed with the command* `showspc`, *see Chapters 2, Chapter 3, and the Qsyn Reference Guide.*

1.4 The robust control problem

The *robust control problem* is defined as the problem to find a feedback compensator $G(s)$ and where appropriate a prefilter $F(s)$, such that the closed loop specifications are satisfied for all plant cases.

Anyone who has tried to design, e.g. with the help of a Bode or Nichols diagram, one $G(s)$ simultaneously for (1.1) knows that the task is very hard or impossible, even when the number of plant cases is very small. The great contribution of Horowitz and Sidi (1972) is that they transformed the *simultaneous* design problem, to a *conventional* feedback design problem for *one* plant only, the nominal plant $P_{\text{nom}}(s)$, with *frequency dependent constraints in the complex plane* on $G(j\omega)$, or equivalently on the nominal open loop $L_{\text{nom}}(j\omega) = P_{\text{nom}}(j\omega)G(j\omega)$. The constraints are called *Horowitz bounds* or *bounds*, and reflect the "interaction" between the size of the plant uncertainty and the "tightness" of the closed loop specifications in a way explained below in Section 1.5.

When the plant is uncertain, it is clear that that the closed loop with a linear feedback compensator $G(s)$ will also remain uncertain, but hopefully less uncertain, at least in the relevant frequency range. The prefilter $F(s)$ is assumed perfectly known and does not contribute to the closed loop uncertainty.

Therefore, a QFT feedback compensator $G(s)$ is designed such that the remaining closed loop uncertainty falls within specifications, and that the closed loop disturbance and noise rejections satisfy their specifications. With $G(s)$ determined, the prefilter $F(s)$ is found such that the closed loop transfer function $Y(s)/R(s)$ be shaped according to specifications.

1.5 The Horowitz bounds

Since $F(s)$ carries no uncertainty, the specification (1.8) for the frequency ω_k implies that the complex number $G(j\omega_k)$ must be chosen such that

$$\max_i \left| \frac{G(j\omega_k)P_i(j\omega_k)}{1+G(j\omega_k)P_i(j\omega_k)} \right| \Big/ \min_i \left| \frac{G(j\omega_k)P_i(j\omega_k)}{1+G(j\omega_k)P_i(j\omega_k)} \right| \le b(\omega_k)/a(\omega_k) \qquad (1.10)$$

with $P_i(j\omega_k) \in \{P_i(j\omega_k)\}$, the plant template at ω_k. (1.10) is often called the *tolerance specification* at ω_k. The tolerance is often given in dB, as $20\log_{10}\left(b(\omega_k)/a(\omega_k)\right)$ = $20\log_{10}\left(b(\omega_k)\right)$ − $20\log_{10}\left(a(\omega_k)\right)$. Clearly if $|G(j\omega_k)|$ is large enough, then (1.10) is satisfied. If, on the other hand, $G(j\omega_k) = -1/P_i(j\omega_k)$ for some i, then (1.10) will not be satisfied. Hence there exists a border, $B_G(\omega_k)$, in the complex $G(j\omega_k)$-plane between satisfactory $G(j\omega_k)$ values, and forbidden $G(j\omega_k)$ values. $B_{TG}(\omega_k)$ is called the *Horowitz bound* for $G(j\omega_k)$ with respect to specification (1.8) or (1.10), where the index T stands for tolerance.

Multiplying $B_{TG}(\omega_k)$ by $P_{\text{nom}}(j\omega_k)$ yields $B_{TL}(\omega_k)$, the Horowitz bound for the nominal open loop $L_{\text{nom}}(j\omega_k) = P_{\text{nom}}(j\omega_k)G(j\omega_k)$ with respect to specification (1.8) or (1.10). The latter bound is most commonly used, since in general it is easier to interactively design by loop shaping the nominal open loop frequency function rather than the feedback compensator transfer function.

In a similar way one finds the Horowitz bounds with respect to the other specifications, e.g. the sensitivity bound $B_{SL}(\omega_k)$ for $L_{\text{nom}}(j\omega_k)$ with respect to the sensitivity specification (1.5).

It is important to note that if

$$\max_i \left|P_i(j\omega_k)\right| \Big/ \min_i \left|P_i(j\omega_k)\right| \le b(\omega_k)/a(\omega_k) \quad \forall \omega_k \tag{1.11}$$

there is no need for a feeback compensator $G(s)$ *with respect to the servo specification* (1.8). For that specification it suffices to design a prefilter $F(s)$ such that

$$a(\omega) \le \left|F(j\omega)P(j\omega)\right| \le b(\omega) \tag{1.12}$$

Example 1.2. Tolerance bound

An example of a tolerance bound $B_{TL}(0.5)$ emanating from the specification given in Figure 1.5 and (1.10), and the uncertain plant of Example 2.1 in Chapter 2 is found in Figure 1.6.

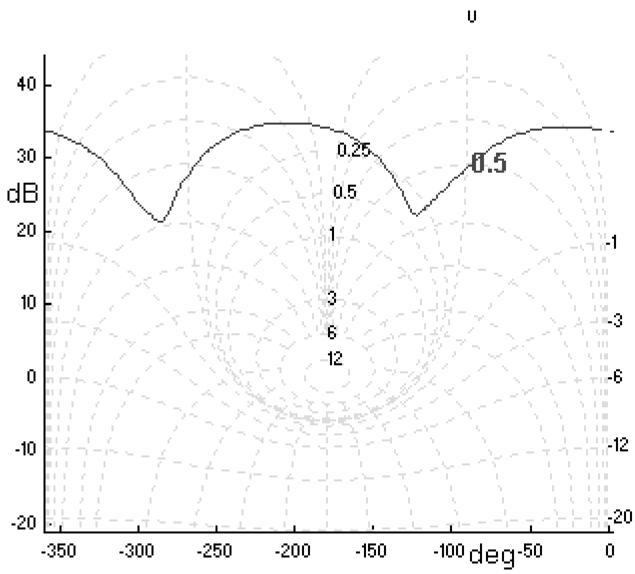

Figure 1.6: A Horowitz bound, $B_{TL}(0.5)$, for the nominal open loop, $L_{\text{nom}}(0.5j)$, emanating from Example 2.1 in Chapter 2. $G(s)$ must be designed such that $L_{\text{nom}}(0.5j)$ will lie above $B_{TL}(0.5)$ in order to have the specification satisfied. The bound was generated by the Qsyn command: `cbnd('ex2_1a','rsrs');`

The bound specifies that the nominal open loop $L_{\text{nom}}(0.5j)$ must be shaped, by an appropriate choice of $G(s)$, such that the point $L_{\text{nom}}(0.5j)$ falls above $B_{TL}(0.5)$ in the Nichols chart in Figure 1.6. It is implicitly clear that the part of the complex plane below $B_{TL}(0.5)$ is forbidden, since the the instability point -1 lies there.

1.6 Feedback compensator design and nominal loop shaping

When all Horowitz bounds for $L_{\text{nom}}(j\omega_k)$ have been computed, for all desired frequencies $\{\omega_k\}$, and for all specifications, and collected in one or more bounds files, the stage is set for the shaping of the nominal open loop $L_{\text{nom}}(j\omega) = P_{\text{nom}}(j\omega)G(j\omega)$, such that for each ω_k,

$L_{\mathrm{nom}}(j\omega_k)$ falls on the permitted side of all of its Horowitz bounds. Loop shaping is done by selecting appropriate first and second order factors to be included in $G(j\omega)$.

Conventional loop shaping is done in a Bode diagram. Every control engineer should be familiar with the gain and phase effect in a Bode diagram of a pure gain, k, a lead, $(1+s/b)/(1+s/(bN))$, a lag, $M(1+s/a)/(1+s/(a/M))$, a PI, $a(1+s/a)/s$, first and second order low pass filters $1/(1+s/a)$ and $1/\left(\left(s^2/\omega_0^2\right)+2\varsigma(s/\omega_0)+1\right)$, respectively, and a notch filter, $\left(\left(s^2/\omega_0^2\right)+2\varsigma_n(s/\omega_0)+1\right)/\left(\left(s^2/\omega_0^2\right)+2\varsigma_d(s/\omega_0)+1\right)$, $\varsigma_n<\varsigma_d$. In addition to these filters, it is useful in QFT to use the "inverse" notch filter, $\left(\left(s^2/\omega_0^2\right)+2\varsigma_n(s/\omega_0)+1\right)/\left(\left(s^2/\omega_0^2\right)+2\varsigma_d(s/\omega_0)+1\right)$, $\varsigma_n>\varsigma_d$, the second order lead/lag $\left(\left(s^2/\omega_n^2\right)+2\varsigma_n(s/\omega_n)+1\right)/\left(\left(s^2/\omega_d^2\right)+2\varsigma_d(s/\omega_d)+1\right)$, and sometimes filters including Right Half Plane (unstable) zeros.

A more fundamental difference between conventional loop shaping and QFT is the fact that in QFT, the nominal open loop cannot be shaped in a Bode diagram, since, for each frequency, the constraining Horowitz bounds are curves in the complex plane. Hence one should use the Nichols diagram, in which to display $L_{\mathrm{nom}}(j\omega)$ and its Horowitz bounds. To become a successful QFT designer, one has to become proficient in Nichols diagram loop shaping, and have the "feeling" how the different filters, and the size of their parameters affect the $L_{\mathrm{nom}}(j\omega)$-curve.

Loop shaping is ofter perceived as difficult and strenous. In order to acquaint the user with loop shaping in standard Matlab, and to make her appreciate the ease with which loop shaping is done in Qsyn (see Section 2.5), we present the beginning of a suggested loop shaping exercise in form of a script m-file. To make the graphics nice, the Qsyn commands `hngrid` and `mgrid` are used for the display. Beginning from an integrator plant, the gain and phase effect of a lead network is appreciated in a Nichols chart, see Figure 1.7. We notice how much the phase is increased in the active frequency range of the lead, and how much, and for which frequencies, the gain is increased. The lead is used to increase the phase in a desired frequency interval, but the price is unfortunately that the gain increases for the same and higher frequencies.

The user is invited to change the parameter values of the lead filter, and study the effect, and to exchange the lead for one or more of the compensation networks mentioned above. The work with this m-file is somewhat tedious. The user will certainly appreciate the convenience in using Qsyn for loop shaping!

Exercise 1.1. Loop shaping

```
% A fresh Nichols chart. Do not forget the zoom option
clg, hold off, hngrid,mgrid(12,10),hold, hzoom

% The frequency vector [rad/s]
w=logspace(-2,3,150);s=j*w;

% The integrator plant
P=1./s;
plot((180/pi)*angle(P), 20*log10(abs(P)),'r')

% Indicate a number of frequencies on the frequency function
wt=[0.01 0.02 0.05 0.1 0.2 0.5 1 2 5 10 20 50 100 200 500 1000];
st = j*wt;
Pt =1./st;
plot((180/pi)*angle(Pt), 20*log10(abs(Pt)),'r*')
```

```
for i=1:16,
  text((180/pi)*angle(Pt(i)),20*log10(abs(Pt(i))),num2str(wt(i))),
end

% A lead network
b=1; a=10; Glead=(1+s/b)./(1+s/a); Gleadt=(1+st/b)./(1+st/a);

% The open loop
L = P.*Glead; Lt = Pt.*Gleadt;
plot((180/pi)*angle(L), 20*log10(abs(L)),'b')
plot((180/pi)*angle(Lt), 20*log10(abs(Lt)),'*b')
for i=1:16, text((180/pi)*angle(Lt(i)), ...
      20*log10(abs(Lt(i))),num2str(wt(i))),end

%Press the zoom button, and zoom in. See figure 1.7

%Change the lead parameters, and study the effect.
%Study the effect of other compensator networks.
```

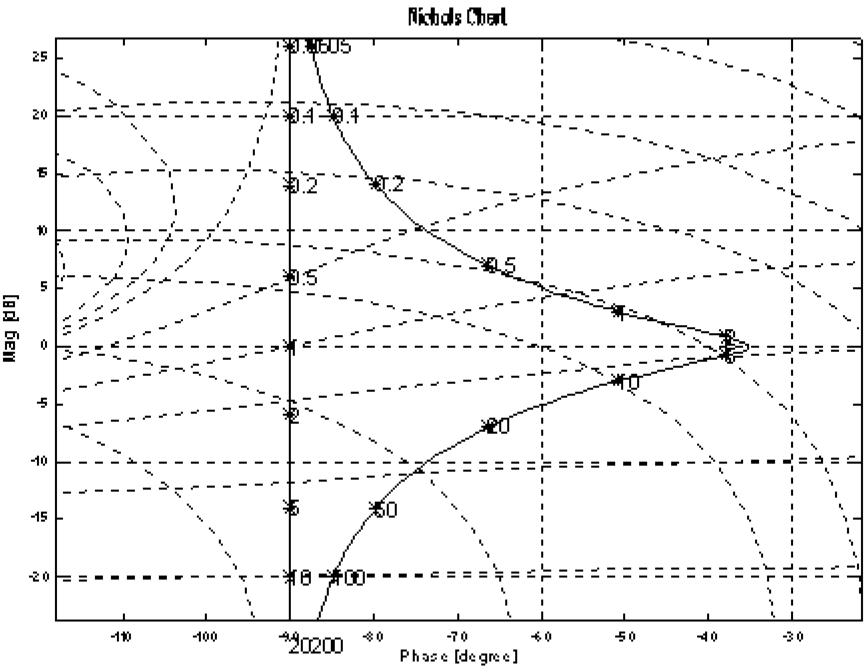

Figure 1.7. The open loops $1/s$ *and* $(1/s)\cdot((1+s)/(1+s/10))$ *in a Nichols diagram. The curves are parametrized by the frequency* [rad/s].

When $G(s)$ has been designed to meet the Horowitz bounds constraints, the design is in general completed by including a first or second order low pass filter with a sufficiently high break off frequency, such that the bounds constraints are still satisfied. The low pass filter makes $G(s)$ strictly proper, and attenuates high frequency measurement noise.

1.7 Closed loop stability

Loop shaping $L_{\text{nom}}(j\omega)$ such that all Horowitz bounds constraints are satisfied does not guarantee stability. In principle one has to check that each open loop case, $L_i(s)$, satisfies the Nyquist criterion, $\Delta\arg(1+L_i(s)) = -p_i$, when $s \in \gamma$, a curve encircling the Right Half Plane in the positive direction, and p_i equals the number of open loop Right Half Plane poles of $L_i(s)$. However, the following theorem covers an important class of plants:

Theorem 1.1. Closed loop stability for plants with simply connected templates

Consider the class of plants described by rational transfer functions, with or without a pure delay. Assume that all plant cases have equal high frequency gain sign, $\mathrm{sign}\left(\lim_{s\to\infty} P_i(s)\right)=\mathrm{sign}\left(\lim_{s\to\infty} P_k(s)\right)$, for all i, k, noting that a pure time delay does not influence the high frequency gain sign. Assume that all plant templates $\{P_i(j\omega)\}$ for all ω are simply connected sets in the *expanded* Nichols chart, i.e. the Nichols chart that spans over multiple Riemann surfaces. Assume that the nominal plant for each frequency belongs to the template, $P_{\mathrm{nom}}(j\omega)\in\{P_i(j\omega)\}$. Assume further that the feedback compensator $G(s)$ has been designed such that the nominal open loop $L_{\mathrm{nom}}(j\omega)=P_{\mathrm{nom}}(j\omega)G(j\omega)$ resides, for each ω, on the permitted side of its Horowitz bound that emanates from a specification that excludes that an open loop template point equals -1. Then, each open loop case $L_{\mathrm{i}}(j\omega)$ satisfies the Nyquist criterion, if and only if $L_{\mathrm{nom}}(j\omega)$ satisfies the Nyquist criterion.

Proof

An outline of the proof is given in the Appendix of Chapter 1.

Remarks

1. The sensitivity specification (1.5) satisfies the assumption of the theorem.
2. The theorem holds also when there are plant cases with a different number of Right Hand Side poles.

●

The implication for the user of QFT is that she should be very careful when she chooses $P_{\mathrm{nom}}(s)$ outside the plant uncertainty set or when the plant templates are not simply connected. The latter may happen when there are plant cases with a different number of free integrators (Gutman *et al* 1994), or when the plant uncertainty set is defined by a finite number of discrete plant cases (Gutman and Nordin, 1995).

1.8 Prefilter design

After the design of the feedback compensater, one closes the loop to get the closed loop uncertainty set, $\left\{G(s)P_i(s)/\left(1+G(s)P_i(s)\right)\right\}$. Computationally, the loop closure is, for each frequency ω_k, an operation on each member of the template $\left\{P_i(j\omega_k)\right\}$. In Qsyn, the command `tplfop` is used.

Now $\left\{\left|G(j\omega_k)P_i(j\omega_k)/\left(1+G(j\omega_k)P_i(j\omega_k)\right)\right|\right\}$ is displayed in Bode's gain diagram, and compared with the specification (1.8). Since the feedback compensator design step ensured that (1.10) is satisfied, there exists a rational, asymptotically stable prefilter $F(s)$ such that (1.8) is satisfied.

It is very easy to loop shape $F(s)$ in a Bode gain diagram. The user who wants to train such loop shaping in the standard Matlab environment may complement Exercise 1.1 by the following lines, and study the gain effect of various filters in another figure window. In Qsyn, loop shaping in the Bode diagram is easily done with the command `fdesign`, see Section 2.7.

```
%Bode gain diagram
figure
semilogx(w,20*log10(abs(P)),'r',w,20*log10(abs(L)),'b'),grid
```

1.9 Simulations of the closed loop

For a sufficiently large number of plant cases, the closed loop is simulated in the frequency domain with respect to the specifications originally given in that domain, e.g. (1.5). The simulation is done also for frequencies not in $\{\omega_k\}$, in order to ascertain that the specifications are satisfied for "all" frequencies. If a specification is not satisfied for a frequency not belonging to $\{\omega_k\}$, it is advisable to include it in a revised set $\{\omega_k\}$, and redo the feedback design.

The closed loop is finally simulated to check that the original time domain specifications are satisfied. Any simulation program can be used.

If all the frequency domain specifications were satisfied during the feedback compensator, and prefilter design stages, and some time domain specification is found to be unsatisfied, It is an indication that the translation of the time domain specifications to the frequency domain was not appropriate. One of the following actions may provide a remedy:

1. Check if changing the prefilter will solve the problem.
2. You could try to "tighten" the current frequency domain specifications, e.g. decrease the tolerance specification (1.10) or decrease the sensitivity specification (1.5), and then redo the design. As a result, the feedback compensator will have higher gain, and the complementary sensitivity function $\{G(s)P_i(s)/(1+G(s)P_i(s))\}$ will have higher bandwidth.
3. Translate the time domain specifications into frequency domain specifications anew, using a more realistic class of transfer functions to approximate the closed loop system. E.g., if it is known that the closed loop will include a resonance, model it. Then redo the design.

It should on the other hand be clear that a (slight) violation of those Horowitz bounds that emanate from translated time domain specifications often yields a closed loop system that satisfies the time domain specifications.

1.10 When your design attempt fails ...

The critical design stage is usually to select the feedback compensator such that the Horowitz bound constraints are satisfied.

If your attempt to design $G(s)$ fails to satisfy one or more Horowitz bounds emanating from translated time domain specifications, you should, in particular if the violation is minor, still go ahead and close the loop, design the prefilter $F(s)$, and simulate the closed loop. You might be lucky ...

If, however, after repeated attempts, including retranslations of the time domain specifications and redesigns of $G(s)$, you fail to satisfy one or more specifications, the reason might be that either you are a cludsy designer (ask one of your more experienced collegues for help), or there is no solution to the design problem.

Unfortunately, it is in general impossible to know *a priori* if the design problem has a solution, except in trivial situations. The following theorem is known in the QFT literature for a long time (see e.g. Horowitz 1992, or Astrom *et al* 1988):

Theorem 1.2. Arbitrary small tolerance and sensitivity

Assume that the plant uncertainty structure is such that all plants have the same high frequency gain sign, and that

$$P_i(s) \to k_i / s^{d_i}, \quad k_i \in [\underline{k}, \overline{k}], \quad d_i \in [\underline{d}, \underline{d}+1, \ldots, \overline{d}-1, \overline{d}] \qquad \text{for all } i \tag{1.13}$$

Then any tolerance specification (1.10), or sensitivity specification (1.5) with $x(\omega) = 1 + \varepsilon, \;\; \varepsilon > 0$, for $\omega \geq \omega_H$, where ω_H is arbitrary, is achievable with a strictly proper feedback compensator $G(s)$.

Proof

An outline of the proof is given in the Appendix of Chapter 1.

Remarks

1. For real systems, the theorem is useless, since it presupposes the possibility of arbitrary high bandwidth. In reality, bandwidth is always limited by actuator saturation, or by the inevitable fact that every real plant includes very large continuous phase uncertainty at high frequency, e.g. modelled with the help of delays,

$$P_i(s) \to k_i e^{-\tau_i s} / s^{d_i}, \quad \tau_i \in [\underline{\tau}, \overline{\tau}], \quad k_i \in [\underline{k}, \overline{k}], \quad d_i \in [\underline{d}, \underline{d}+1, \ldots, \overline{d}-1, \overline{d}] \quad \text{for all } i \tag{1.14}$$

   or with the help of unstructured uncertainty (1.3)

$$P_i(s) \to k_i (1 + M(s)) / s^{d_i}, \quad k_i \in [\underline{k}, \overline{k}], \quad d_i \in [\underline{d}, \underline{d}+1, \ldots, \overline{d}-1, \overline{d}] \tag{1.15}$$

   with $1 - |M(j\omega)|$ being a sufficiently small positive number for high frequencies. It is recommended that the QFT user either states a bandwidth limitation explicitly, or includes delay or unstructured undertainty in his high frequency uncertainty description.
2. Although the theorem is useless, it gives an indication to the user, that the loop might be closed at a complementary sensitivity function bandwidth where the phase uncertainty is sufficiently small. It then remains to check if this bandwidth is sufficient to satisfy the specifications.

●

So, if your friend's design also fails to meet the specifications, one or both of the following actions are recommended, before attempting a redesign:

3. Relax the difficult specifications in those frequency bands where it was impossible to satisfy the corresponding bounds.
4. Get to know your plant better so that the uncertainty is decreased. In particular, try to diminish the templates at those frequencies where the Horowitz bound violations occured.

The last point includes a connection to adaptive control. If it is impossible to gain enough plant knowledge off line to decrease the template sizes, it might be possible to combine robust and adaptive control in such a way that on line identification provides current template estimates to an adaptive QFT controller. Attempts in this direction are reported in Gutman *et al* (1988), and Yaniv, Gutman, and Neumann (1990).

Appendix to Chapter 1

Proof (outline) for Theorem 1.1

It is well known that for simultaneous stabilizability, the sign of the high frequency gain must be equal for all plant cases. Moreover, for the class of considered plant, closed loop stability implies that the Nyquist curve passes on the same side of -1 for all plant cases. Since the Horowitz bound is computed such that $L_i(j\omega) \neq -1$ for every i and ω, and the templates are simply connected, $L_{nom}(j\omega)$ satisfying its Horowitz bounds implies that all plant cases pass on the same side of -1. The details of the proof are left to the user. •

Proof (outline) of Theorem 1.2

The idea behind the proof is that the gain of $G(s)$ is made so large such that the specifications are satisfied. The remaining problem is then to achieve simultaneous stabilization of the multiple integrator plants of the right hand side of (1.13). Such stabilization is always possible by sufficiently many lead networks that add phase around the bandwidth frequency. To make $G(s)$ strictly proper, a low pass filter with a sufficiently high break off frequency is included. It is left to the user to synthesize such a compensater in detail for (1.11). •