



KTH Matematik

**Homework 2: 5B1872: Optimal Control Spring 2006**

**Grading: You may use  $\frac{\text{your credit}}{20}$  extra points on the exam.**

1. Consider the time optimal control problem

$$\min t_f \quad \text{subj.to} \quad \begin{cases} \ddot{y} = u, & |u| \leq 1, & t_f \geq 0 \\ y(0) = \alpha, & y(t_f) = 0 \\ \dot{y}(0) = \beta, & \dot{y}(t_f) \geq 0 \end{cases}$$

An interpretation in terms of the rocket car example is that we want to pass the origin with positive velocity as soon as possible.

- (a) What are the possible optimal switching sequences?
- (b) Sketch the optimal solution in a phase plane plot where the switching curve and the control values should be clearly indicated.

..... 10p

2. A lifeguard is standing on the beach at position  $(0, 0)$  when he discovers a person in distress at position  $(a, b)$ . The equations of motion of the lifeguard are

$$\begin{aligned} \dot{x}(t) &= v(y(t)) \cos(u(t)), & x(0) &= 0, & x(T) &= a > 0 \\ \dot{y}(t) &= v(y(t)) \sin(u(t)), & y(0) &= 0, & y(T) &= b > 0 \end{aligned}$$

where his speed depends on the distance to the beach

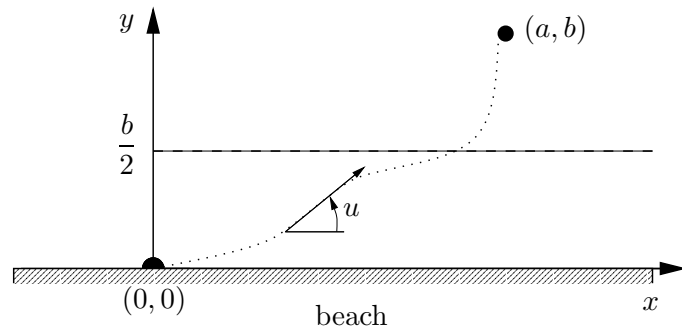
$$v(y) = \begin{cases} 1, & 0 \leq y \leq \frac{b}{2} \\ \frac{1}{2}, & \frac{b}{2} < y \leq b \end{cases}$$

Find the optimal control function  $u(t)$  such that the lifeguard reaches the person in shortest time. Do the numerical calculation of the intersection point with the line  $y = \frac{b}{2}$  for the case when  $b = 4$  and  $a = 2$ . Note that the function  $v(y)$  is not smooth, therefore you have to treat the two regions separately and then combine using, for example, dynamic programming.

..... (10p)

3. Determine the bang-bang control for the following time optimal control problem

$$\min T \quad \text{subj. to} \quad \begin{cases} \dot{x} = x^2 - \frac{1}{4} - xu \\ x(0) = \frac{1}{2}, & x(T) = -\frac{1}{2} \\ |u| \leq 1 \end{cases}$$



Figur 1: The lifeguard is positioned at  $(0, 0)$  and wants to reach the person who is fixed at  $(a, b)$  in as short time as possible.

You are allowed to compute the switching time numerically.

*Hint: Study the switching function. Use that the solution of a differential equation  $\dot{\lambda}(t) = g(t)\lambda(t)$  has constant sign.*

..... (10p)