Let \( f_1, \ldots, f_m \) be real valued functions defined on a given subset \( X \) of \( \mathbb{R}^n \), and consider the following so called minimax-problem:

\[
P : \text{minimize } \max_i \{ f_i(x) \} \tag{1}
\]

For a given \( x \in X \), the entity \( \max_i \{ f_i(x) \} \) is the largest of the \( m \) real numbers \( f_1(x), \ldots, f_m(x) \).

Using the extra variable \( z \), \( P \) can be formulated on the equivalent form

\[
P : \text{minimize } z \tag{2}
\]

\[
s.t. \quad f_i(x) - z \leq 0, \quad i = 1, \ldots, m, \quad z \in \mathbb{R} \text{ and } x \in X,
\]

or on the more compact, but still equivalent, form

\[
P : \text{minimize } z \tag{3}
\]

\[
s.t. \quad f(x) - e^T z \leq 0, \quad z \in \mathbb{R} \text{ and } x \in X,
\]

where \( f(x) = (f_1(x), \ldots, f_m(x))^T \) and \( e = (1, \ldots, 1)^T \).

For a given vector \( y \in \mathbb{R}^m \) with non-negative components the corresponding Lagrange relaxed problem is given by

\[
PR_y : \text{minimize } z + y^T (f(x) - e^T z) \tag{4}
\]

\[
s.t. \quad z \in \mathbb{R} \text{ and } x \in X.
\]

Here the objective function has the equivalent form \( (1 - e^T y) z + y^T f(x) \).

The dual objective function \( \varphi(y) \) is given by the optimal value to the problem \( PR_y \).

But if \( e^T y \neq 1 \), then \( (1 - e^T y) z \) is not bounded from below, since \( (1 - e^T y) z \rightarrow -\infty \) if \( e^T y > 1 \) and \( z \rightarrow +\infty \) or if \( e^T y < 1 \) and \( z \rightarrow -\infty \).

Therefore, we only need to consider the \( y \in \mathbb{R}^m \) that satisfy \( y \geq 0 \) and \( e^T y = 1 \), and for these \( y \) the dual objective function is given by \( \varphi(y) = \min_{x \in X} \{ y^T f(x) \} \), where for simplicity we assume that the minimal value exists.

The dual problem is now finally given by

\[
D : \text{maximize } \varphi(y) \tag{5}
\]

\[
s.t. \quad e^T y = 1 \text{ and } y \geq 0.
\]