Computer Exercise
Mathematical Systems Theory, SF2832
Fall 2007

Satisfactory completion of this computer exercise gives three (3) bonus credits for this year's final exam. The examination of the computer exercise is *oral and written*. Firstly the results should be presented in a written report (use a word processor) and secondly each student should be prepared to answer question about the exercise when it is handed back. Hand in the report at the latest **October 5, 5:00pm**, and pick it up after agreement with your project examiner. Do attach your commented MATLAB code to your report! Cooperation in groups of no more than two students is allowed. Only one report for each group is required.

The necessary MATLAB m-files can be found at:

http://www.math.kth.se/optsyst/grundutbildning/kurser/SF2832/kursPM.html

The intention of this exercise is to exemplify how easy it is to apply the theory of the course using a computer. We will use the "Control System Toolbox" in MATLAB, which will save us lots of programming. Names written with bold font are command names in MATLAB.

Write **help control** to get a list of available functions in the "Control System Toolbox" or use the help browser.

Reachability, observability and pole placement

1. Consider two carts of mass M on wheels which are joined by a spring with spring constant K. The carts move in one line on level ground and are at positions z_1 and z_2 respectively. One pair of wheels on each cart is powered by a DC electric motor of inertia J, torque constant k and resistance R. The powered wheels on cart 1 are at angle θ_1 and the powered wheels on cart 2 are at angle θ_2. r is the ratio of motor torque to the linear force applied to the cart. The system inputs are the voltages e_1 and e_2 on the motors. This system is described by the following differential equations:

$$
\begin{aligned}
M\ddot{z}_1 &= K(z_2 - z_1) + rJ\ddot{\theta}_1 \\
M\ddot{z}_2 &= K(z_1 - z_2) + rJ\ddot{\theta}_2 \\
RJ\ddot{\theta}_1 &= -k^2\dot{\theta}_1 + ke_1 \\
RJ\ddot{\theta}_2 &= -k^2\dot{\theta}_2 + ke_2
\end{aligned}
$$

(a) Write the system in the usual form, that is

$$\dot{x} = Ax + Bu$$

where the state vector is defined as $x = [z_1, z_2, \dot{z}_1, \dot{z}_2, \dot{\theta}_1, \dot{\theta}_2]'$.

(b) Suppose all the parameters are equal to 1. Is the system completely reachable using only one motor? (Use **ctrb**.)

(c) Suppose from now on that both motors are being used. Is the system completely reachable?

(d) Suppose that only z_1 is measured. Is the system observable?   (Use **obsv**.)

(e) Suppose only $\dot{z}_1$ and $\dot{z}_2$ are measured.

- Is the system observable?
- Determine the corresponding transfer function $R(s)$. (Use the commands **ss** and **ss** as **tf(ss(A,B,C,D))**.)
- Is the system a minimal realization of $R(s)$?
- Find a minimal realization of $R(s)$ using **minreal**. Verify that the minimal realization is both reachable and observable.

(f) Suppose sensors are very expensive. Which variables should be measured in order for the system to be completely observable and for us to spend as little money as possible?

(g) Determine if the system is stable,

- first by using the Lyapunov method,   (Use **lyap**)
- secondly by determining the Jordan form of the A-matrix using the command **jordan**. If your computer cannot use the command **jordan** then use the command **eig** and discuss what conclusion you can make.

(h) Stabilize the system with a state feedback by placing the closed loop poles at $\{0, -1, -2, -3, -4, -5\}$.
Use **place** : note the Matlab convention "$A - BK$".
Why can't you place the poles at $\{-1, -2, -3, -4, -5, -6\}$?

The inverted pendulum on a cart

2. In the file "pendulum.m" the matrices A, B, C, D that correspond to the linearized realization of the pendulum with the states

$$x = \begin{bmatrix} \phi & \dot{\phi} & s & \dot{s} \end{bmatrix}'$$

are given.

(a) Is the realization reachable, observerable or minimal ?

(b) Determine whether the system is stable by determining the location of the poles.   (Use e.g. **eig, pole**)

(c) Design three different state feedbacks, all with closed-loop poles in the left half-plane.

  i. All poles close to the origin.
  ii. All poles with small imaginary parts but large real parts.
  iii. All poles with large imaginary parts but small real parts.

(d) Now study the closed loop system by

  i. determining the impulse response, equivalent to a chock on the cart.
    Use **impulse** , and the time vector **t=0:0.05:10** .

    ii. determining the step response, equivalent to moving the cart sideways. Use **step** , and the time vector **t=0:0.1:40** .

   Compare the feedback design from above and explain the differences.

(e) Now let the system be described by $\dot{x} = (A+BK)x+Sw$, where $S = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}'$. We let w describe disturbances on the pendulum, caused by wind and annoying flies.

   Make a random vector of normalized white noise by **w=randn(1,length(t))/10** and simulate the system using **lsim** for the different feedback designs.

Good Luck! Don't hesitate to ask (by email or phone) if anything is unclear.