KTH Matematik

Homework 2
Mathematical Systems Theory, SF2832
Fall 2007
You may use min(3,(your score)/10) as bonus credit on the exam.

1. Consider the pair (C, A), where

$$A = \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Solve the Lyapunov equation $A^T P + PA + C^T C = 0$ and determine whether A is stable or not. .... (8p)

2. You will in this problem derive and investigate a number of realizations for the transfer function

$$R(s) = \begin{bmatrix} \frac{s+2}{s+1} \\ \frac{1}{2s+4} \end{bmatrix}$$

(a) Determine the standard reachable realization of $R(s)$. .... (4p)

(b) Is the realization in (a) observable? .... (1p)

(c) Determine the standard observable realization of $R(s)$. .... (4p)

(c) Is the realization in (c) reachable? .... (1p)

3. Two state space representations (A, B, C) and $\bar{A}, \bar{B}, \bar{C})$ are said to be equivalent if there exists a nonsingular matrix T such that

$$(\bar{A}, \bar{B}, \bar{C}) = (TAT^{-1}, TB, CT^{-1})$$

(a) Are the following two realizations equivalent

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix} \right)$$
$$(\bar{A}, \bar{B}, \bar{C}) = \left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 0.5 \end{bmatrix} \right)?$$

Compute T if they are equivalent. .... (4p) (*b*) Are the following two realizations equivalent

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \end{bmatrix} \right)$$
$$(\bar{A}, \bar{B}, \bar{C}) = \left(\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \end{bmatrix} \right)?$$

Compute T if they are equivalent. . . . . (4p)

4. In this problem we study two different stability concepts for a linear time-varying system.

(*a*) Determine the solution to the homogeneous system

$$\dot{x}(t) = -\frac{1}{2t}x(t), \quad x(1) = x_0, \ t \geq 1$$

. . . . (2p)

(*b*) Is the solution in (*a*) asymptotically stable? . . . . (1p)

(*c*) Is the controlled system

$$\dot{x}(t) = -\frac{1}{2t}x(t) + u(t), \quad x(1) = 0, \ t \geq 1$$

BIBO-stable? . . . . (3p)

5. Let Q be a symmetric and positive definite matrix ($Q > 0$) and α a positive number. Prove that the following are equivalent

(*a*) all eigenvalues of A with magnitude less than $\alpha \in (0, 1)$, i.e. $|\lambda(A)| < \alpha$,

(*b*) there exists a positive definite P such that

$$A^T PA - \alpha^2 P + \alpha^2 Q = 0.$$

. . . . (8p)