KTH Matematik

Homework 3
Mathematical Systems Theory, SF2832
Fall 2007
You may use min(3,(your score)/10) as bonus credit on the exam.

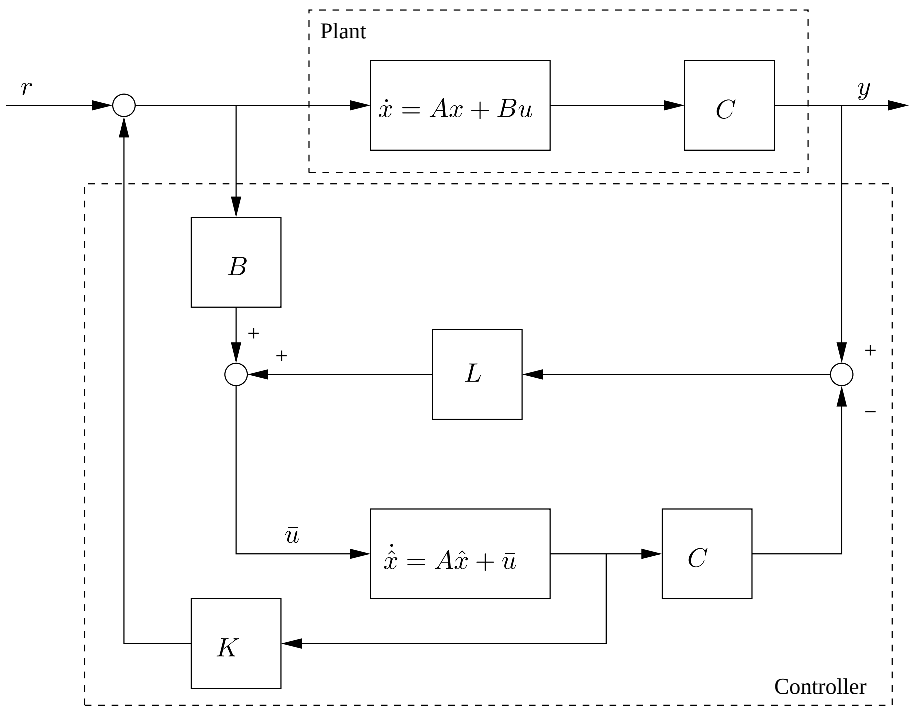
1. Determine a state feedback K such that the eigenvalues of the closed loop system $\dot{x} = (A + BK)x$ are located in $\{-1, -2, -3\}$, for the case when

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

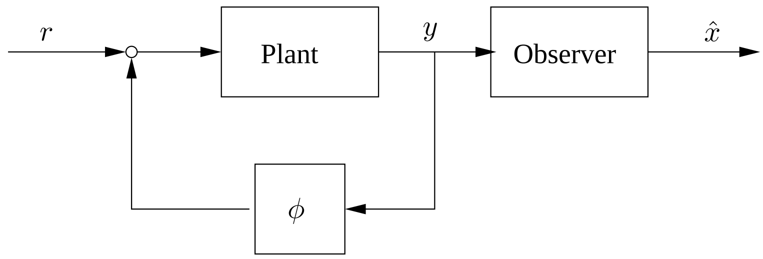
. . . . (4p)

2. In this problem we consider observer based control.

(a) The observer based control system in Figure 1 has a 6^{th} order single-input-single-output plant determined by the matrices (A, B, C). What is the order of the transfer function from r to y, provided that there is no zero pole cancelation?
. . . . (4p)



Figur 1: Observer based control system



Figur 2: Observer for feedback system

(*b*) Under what conditions can the poles of the transfer function from r to y be placed in arbitrary location? . . . . (1p)

(*c*) Consider the case when the plant is

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

Assume that the state is not available. Design an observer based controller that stabilizes the system. Let the closed loop poles be located at $-1, -2$ and let the observer dynamics have both poles at -2. . . . . (5p)

3. Consider a harmonic oscillator affected by additive white Gaussan noise

$$\ddot{z}(t) = -z(t) + v(t)$$
$$y(t) = z(t) + w(t)$$

where $E\{v(t)\} = E\{w(t)\} = 0$, $E\{v(s)v(t)\} = E\{w(s)w(t)\} = \delta(t - s)$, and $E\{v(s)w(t) = 0$. Design the stationary Kalman filter that gives the minimum variance estimate of the position $z(t)$ and its velocity $\dot{z}(t)$.

. . . . (8p)

4. In this problem we will investigate observer design for the feedback system in Fig 2. The plant is assumed to have a minimal state space realization

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

and ϕ is a constant matrix with dimensions consistent with the input and output dimensions for the plant. We will in the problem investigate whether the feedback through the gain ϕ affects our ability to design an observer.

(*a*) Suppose

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, \quad \phi = 1$$

Design an observer with poles in -2. .... (6p)

(b) Show that the necessary and sufficient condition for arbitrary pole placement of the observer is independent of the choice of ϕ.
.... (4p)

5. Consider the discrete time system

$$x(t+1) = Ax(t) + Bv(t),$$
$$y(t) = Cx(t) + Dw(t),$$

where

$$x(0) = x_0, \qquad \mathrm{E}\,[v(t)v'(s)] = \delta_{t,s}I,$$
$$\mathrm{E}\,[x(0)x'(0)] = P_0, \qquad \mathrm{E}\,[w(t)w'(s)] = \delta_{t,s}I,$$

and where $v, w, x(0)$ are uncorrelated with zero mean value.

(a) Determine a recursive equation for the covariance matrix $V(t) = E\{x(t)x(t)'\}$.
.... (2p)

(b) Prove that $V(t)$ converges to a stationary value as $t \to \infty$ if the eigenvalues of A satisfies $|\lambda(A)| < 1$. .... (2p)

(c) Let $\hat{x}(t) = Ex(t)$ be the Kalman estimate defined by

$$\hat{x}(t+1) = A\hat{x}(t) + K(t)(y(t) - C\hat{x}(t))$$

and let $K(t) = AP(t)C'[CP(t)C' + DD']^{-1}$ be the Kalman gain. Determine a recursive equation for the covariance matrix $P(t) = E\{(x(t)-\hat{x}(t))(x(t)-\hat{x}(t))'\}$ of the estimation error. .... (4p)

(d) Show that $R(t) := V(t) - P(t)$ is positive semi-definite for all t .... (2p)