KTH Matematik

Homework 2
Mathematical Systems Theory, SF2832
Fall 2008
You may use min(3,(your score)/10) as bonus credit on the exam.

1. Consider the pair (C, A), where

$$A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

For what a_1 and a_2 the Lyapunov equation $A^T P + PA + C^T C = 0$ has a positive definite solution? ..... (6p)

2. You will in this problem derive and investigate a number of realizations for the transfer function

$$R(s) = \begin{bmatrix} \frac{\gamma}{s+1} & \frac{s+2}{s+1} \\ \frac{1}{s+1} & \frac{s+2}{s+1} \end{bmatrix},$$

where $\gamma > 0$ is a constant.

(a) Determine the standard reachable realization of $R(s)$. ..... (4p)

(b) Is the realization in (a) observable? ..... (2p)

(c) Determine the standard observable realization of $R(s)$. ..... (4p)

3. Suppose the following is a realization of a given $r(s)$:

$$(A, B, C) = \left(\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \right)$$

(a) Is this realization minimal? ..... (3p)

(b) If not, use Kalman decomposition to find a minimal realization. ..... (5p)

4. Consider

$$\dot{x} = Ax + Bu$$
$$y = Cx,$$

where,

$$(A, B, C) = \left(\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \right)$$

(a) Is $x = 0$ asymptotically stable when u is set to zero? ..... (2p)

(b) Is the controlled system BIBO-stable when the initial state is set to zero? (2p)

(b) Discuss if A being a stable matrix is necessary for BIBO stability..... (3p)

5. Consider a time-invariant controllable system

$$\dot{x} = Ax + Bu.$$

Let $W(T) = \int_0^T e^{-A\tau} B B^T e^{-A^T \tau} d\tau$, where T is an arbitrary positive number.

Show that if $u = -B^T W^{-1}(T) x$, then the overall system is asymptotically stable (Hint: consider Lyapunov function $x^T W^{-1} x$). ..... (8p)