KTH Matematik

Homework 3
Mathematical Systems Theory, SF2832
Fall 2008
You may use min(3,(your score)/10) as bonus credit on the exam.

1. Determine a state feedback K such that the eigenvalues of the closed-loop system $\dot{x} = (A + BK)x$ are located in $\{-1, -2, -3\}$, for the case when

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

.... (5p)

2. Consider

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & a \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & -1 \end{bmatrix} x,$$

where a is a constant.

(a) Can we always design a feedback controller $u = kx$ such that the closed-loop poles are placed in $\{-1, -1\}$? .... (3p)

(b) Is the resulting closed-loop system observable? .... (3p)

(c) Assume now that the state is not available. Can we always design an observer based controller that stabilizes the system, with the closed-loop poles located at $\{-1, -1\}$ and the observer dynamics having poles at $\{-2, -2\}$? ...... (4p)

3. Consider

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -ax_1 - 2x_2 + u,$$

and the performance index

$$J = \int_0^\infty (2x_1^2 + x_2^2 + u^2)dt.$$

(a) Find all symmetric ARE solutions in terms of a. .... (4p)

(b) What is the optimal control $u = kx$? .... (3p)

4. Consider the algebraic Riccati equation

$$A^T P + PA - PBB^T P + C^T C = 0.$$

(a) Assume P is a real positive **semidefinite** solution. Show that $kerP$ is A-invariant (i.e, $\forall x \in kerP,\ Ax \in kerP$) and $kerP \subset kerC$. .... (4p)

(b) Show that if (C, A) is observable, then every positive semidefinite solution P is positive definite. **Hint:** use the conclusions in (a) .... (4p)

5. Consider the discretized Newton's system with noise

$$\begin{aligned} x_1(t+1) &= x_1(t) + Tx_2(t) \\ x_2(t+1) &= v(t) \\ y(t) &= x_1(t) + w(t), \end{aligned}$$

where $T > 0$, $E\{v(t)\} = E\{w(t)\} = 0$, $E\{v(s)v(t)\} = E\{w(s)w(t)\} = 2\delta_{ts}$, and $E\{v(s)w(t)\} = 0$. Design the Kalman filter for the system.

.... (5p)