KTH Matematik

Homework 3
Mathematical Systems Theory, SF2832
Fall 2009
You may use min(3,(your score)/10) as bonus credit on the exam.

1. Determine a state feedback K such that the eigenvalues of the closed-loop system $\dot{x} = (A + BK)x$ are located in $\{-1, -2, -3, -4\}$, for the case when

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

. . . . (5p)

2. Consider

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x.$$

(*a*) Is the system observable? . . . . (1p)

(*b*) Design a feedback controller $u = kx$ such that the closed-loop poles are placed in $\{-1, -1\}$. . . . . (2p)

(*c*) Is the resulting closed-loop system observable? Why? . . . . (2p)

(*d*) Assume now that the state is not available. Can we always design an observer based controller that stabilizes the system, with the closed-loop poles located at $\{-1, -1\}$ and the observer dynamics having poles at $\{-1, -1\}$? . . . . . . (2p)

3. Consider

$$\dot{x}_1 = ax_1 + x_2$$
$$\dot{x}_2 = x_1 + ru,$$

where $a > 0$, $r > 0$. Given the cost function

$$J = \int_0^\infty (x_2^2 + u^2)dt,$$

let $u = -B^T P x$ denote the optimal control.

- What are the eigenvalues of $(A - BB^T P)$ as $r \to \infty$? . . . . (5p)

4. Consider the algebraic Riccati equation

$$A^T P + PA - PBB^T P + C^T C = 0.$$

- Assume P is a real positive **semidefinite** solution. Show that if (C, A) is observable, then $(A - BB^T P)$ is a stable matrix, i.e. all eigenvalues have negative real parts. .... (6p)

5. All conclusions about Kalman filter still hold if we replace $\mathcal{E}\{w(t)w^T(t)\} = R > 0$ by $\mathcal{E}\{w(t)w^T(t)\} = R(t) > 0$. Namely allow the covariance matrix for the noise to be time-varying.

Now consider the problem of measuring some constant scalar quantity x. Suppose initially nothing is known about x (i.e. $P(0) = \infty$). Then at each time instance $t = 0, 1, \cdots, n$, a measurement of x, $y(t)$, is made, with error covariance $r(t)$.

(a) Express the optimal estimation of x at t, $\hat{x}(t)$, which is based on measurements up to $t - 1$, by Kalman filter. .... (3p)

(b) Show in the Kalman filter, $P(t + 1) < P(t)$. .... (2p)

(c) Write down the expression of $P(t)$ in terms of $r(i)$, $i = 0, 1, \cdots, t - 1$. .... (2p)