KTH Matematik

Homework 2
Mathematical Systems Theory, SF2832
Spring 2011
You may use min(5,(your score)/7) as bonus credit on the exam.

1. Consider the pair (C, A), where

$$A = \begin{bmatrix} a_1 & a_2 \\ -a_2 & a_1 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

(a) For what a_1 and a_2 the Lyapunov equation $A^T P + PA + C^T C = 0$ has a positive definite solution? . . . . . . (2p)

(b) Find the positive definite solution P. . . . . . . (3p)

2. This problem concerns stability as well.

(a) For $b = [1\ 1]^T$ and $c = [1\ 1]$, construct a 2×2 matrix A that is not asymptotically stable, but the resulting system is still BIBO stable . . . . . . (2p)

(b) Let $\dot{x} = Ax$, where

$$A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}.$$

When is the system stable, but not asymptotically stable? . . . . . . (1p)

(c) Show that all the eigenvalues of A have real parts less than $-r < 0$ if and only if for a positive definite N

$$A^T P + PA = -N - 2rP$$

has a positive definite solution P. . . . . . . (2p)

3. Consider

$$R(s) = \begin{bmatrix} \dfrac{k}{s+1} & \dfrac{1}{(s+1)^2} \\ \dfrac{2}{s+1} & \dfrac{1}{(s+1)^2} \end{bmatrix},$$

where k is a constant.

(a) Determine the standard reachable realization of $R(s)$. . . . . . . (3p)

(b) Is the realization in (a) observable? . . . . . . (2p)

(c) Determine the standard observable realization of $R(s)$. .... (3p)

(d) What is the McMillan degree of $R(s)$? .... (2p)

4. Suppose the following is a realization of a given $r(s)$:

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & -6 & -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} c_1 & c_2 & 1 & 0 \end{bmatrix} \right)$$

(a) Show that the set of values of (c_1, c_2) such that the realization is not minimal is a line on the (c_1, c_2) plane. ....(4p)

(b) Is $(c_1, c_2) = (1, 2)$ on the line? If so, use Kalman decomposition to find a minimal realization. .... (6p)

5. Consider a time-invariant controllable system

$$\dot{x} = Ax + Bu,$$

where all the eigenvalues of A have positive real parts. Let $W = \int_0^\infty e^{-A\tau} BB^T e^{-A^T\tau} d\tau$. Show that for $u = -kB^T W^{-1} x$, $k > \frac{1}{2}$, the closed-loop system is asymptotically stable. .... (5p)