KTH Matematik

**Homework 1**
**Mathematical Systems Theory, SF2832**
**Spring 2012**
**You may use min(5,(your score)/5) as bonus credit on the exam.**

**1.** Find the state transition matrix for the following systems

(a) $\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & t \end{bmatrix} x(t)$

..... (2p)

(b) $\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2k & -2-3k & -3-k \end{bmatrix} x(t),$

where the constant $k > 2$. ..... (3p)

(c) Let

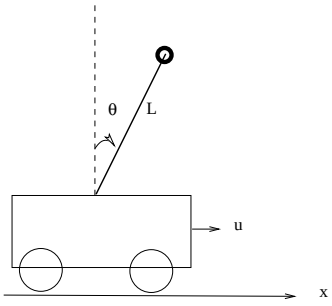
$$A = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix},$$

where $a_1, a_2, a_3$ are constants satisfying $a_1^2 + a_2^2 + a_3^2 = 1$. Show

$$e^{At} = I + A\sin(t) + A^2(1 - \cos(t)).$$

(Hint: show first $A^{2n+1} = (-1)^n A,\ n \geq 0$ and $A^{2n} = (-1)^{n+1}A^2,\ n \geq 1$.) (4p)

**2.** Consider the inverted pendulum




Figur 1: Inverted pendulum on a cart.

The following equation describes the motion of the pendulum around the equilibrium $\theta = 0$:

$$L\ddot{\theta} - g\sin(\theta) + \ddot{x}\cos(\theta) = 0$$

(*a*) We consider $\ddot{x}$ as the input $u$ and $\theta$ as the output $y$. Let $x_1 = \theta$ and $x_2 = \dot{\theta}$. Derive the state space model for the linearized system (i.e. let $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$). (2p)

(*b*) Derive the input-output description of the system in (a), with $L = 1$ and $g = 10$. (3p)

(*c*) Show the model you derive in (a) is both controllable and observable. ... (4p)

**3.** Consider the attitude control system for a spacecraft (after linearization) :

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

$$\begin{pmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{pmatrix} = \sum_{i=1}^{m} b_i u_i,$$

where $\phi$, $\theta$, $\psi$ describe the angular position of the spacecraft, and $\omega_i$ are angular velocities. Each scalar control $u_i$ is implemented by an opposing pair of gas jets. $b_i$ depends on where the gas jets are placed.

(*a*) Suppose we have three control inputs. Discuss conditions on the placements of the gas jets (i.e. conditions on $b_i$) such that the system is controllable. (4p)

(*b*) When only two pairs of gas jets are functioning, can the system still be controllable? (3p)