KTH Matematik

Homework 2
Mathematical Systems Theory, SF2832
Spring 2012
You may use min(5,(your score)/7) as bonus credit on the exam.

1. (*a*) It is well known that (C, A), where A is $n \times n$, is observable if and only if the complex matrix

$$\begin{bmatrix} sI - A \\ C \end{bmatrix}$$

has rank n for all $s \in \mathcal{C}$. Here show only the necessity of this rank condition for observability. .... (3p)

(*b*) The purpose of this problem is to prove the statement on zero-pole cancellation on p. 50 of the compendium. Given the following control system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} c_1 & c_2 \cdots & c_n \end{bmatrix} x,$$

show the system is observable if and only if no root of the polynomial $c_n s^{n-1} + \cdots + c_1$ is an eigenvalue of the A matrix (hint: use the criterion in (a))... (4p)

(*c*) For the system in (b), discuss conditions on c such that $(c, A + bk)$ is observable for arbitrary k. .... (2p)

2. Consider a time-invariant system

$$\dot{x} = Ax,$$

where $x \in R^n$ and $A^T = -A$, namely A is skew symmetric.

Show

(*a*) The system is not asymptotically stable (around $x = 0$). .... (2p)

(*b*) The system is (critically) stable. .... (2p)

3. (*a*) Consider the system in 1.(b) and let $n = 2$. Using state feedback one can place the poles of the closed-loop system at $s_1 = -p$ and $s_2 = -2p$, where $p > 0$, and let p be sufficiently large. This is the so-called high gain control. Show that however, there are closed-loop solutions $x(t)$, such that as $p \to \infty$, $|x(T_p)| \to \infty$, for some finite T_p. .... (3p)

(b) Consider

$$\dot{x} = Ax + Bu$$
$$y = Cx.$$

Show that the reachable and unobservable subpaces are invariant under output feedback control $u = Ky + v$. .... (3p)

4. Consider

$$R(s) = \begin{bmatrix} \frac{s+2}{s+1} & \frac{1}{s+1} \\ \frac{2}{s+1} & \frac{s+1}{s+2} \end{bmatrix},$$

(a) Determine the standard reachable realization of $R(s)$. .... (3p)

(b) Is the realization in (a) observable? .... (2p)

(c) Determine the standard observable realization of $R(s)$. .... (3p)

(d) What is the McMillan degree of $R(s)$? .... (2p)

5. Suppose the following is a realization of a given $R(s)$:

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \right)$$

(a) Find a feedback control $u = Kx$ that assigns poles to $\{-1, -1, -2, -2\}$. . (3p)

(b) Is the realization minimal? If not, use Kalman decomposition to find a minimal realization. .... (3p)