KTH Matematik

Homework 3
Mathematical Systems Theory, SF2832
Fall 2013
You may use min(5,(your score)/4) as bonus credit on the exam.

1. Determine a state feedback K such that the eigenvalues of the closed-loop system $\dot{x} = (A + BK)x$ are located in $\{-1, -1, -2, -2\}$, for the case when

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

.... (3p)

2. Consider a state space realization (A, b, c) as follows

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} c_1 & c_2 & 1 \end{bmatrix} x,$$

where $c_1,\ c_2$ are constants.

(a) For what $c_1,\ c_2$ is the system observable? .... (1p)

(b) Design a feedback controller $u = kx$ such that dimension of the unobservable subspace for $(c, A + bk)$ is maximized while $A + bk$ has at least one eigenvalue at -1. .... (3p)

(c) Assume now that $c_1 = 2,\ c_2 = 3$ and the full state is not available. Can we always design an observer-based control that stabilizes the overall system, with the closed-loop poles located at $\{-1, -2, -3\}$ and the observer dynamics having poles at $\{-1, -2, -3\}$? .... (2p)

3. Consider a state space system (A,b) as follows

$$\dot{x}_1 = x_1 + ax_2$$
$$\dot{x}_2 = ax_1 + u,$$

where a is a constant, and the cost function

$$J = \int_0^{t_1} (x_2^2 + \epsilon^2 u^2) dt.$$

Assume $x^*(t)$ is the optimal trajectory for a given initial point $(x_1(0)\ x_2(0))^T$ with the optimal control $u = -\epsilon^{-2} b^T P(t) x(t)$.

(a) For what a is $P(t)$ positive definite $\forall t < t_1$?. .... (2p)

(b) Now let $t_1 = \infty$. For the case where P_∞ is positive definite, compute the eigenvalues of $A - \epsilon^{-2}bb^T P_\infty$ as $\epsilon \to 0$. .... (3p)

4. Consider a one-dimensional system

$$\begin{aligned} x(t+1) &= ax(t) + v(t) \\ y(t) &= x(t) + w(t), \end{aligned}$$

where $a \neq 0,\ v, w$ are uncorrelated white noises, with covariances $\sigma,\ r$ respectively.

(a) Design a Kalman filter $\hat{x}(t)$ for $x(t)$. .... (2p)

(b) Express the covariance matrix $p(t) = E\{(x(t) - \hat{x}(t))^2\}$ in terms of $a, \sigma,\ r$.(2p)

(c) What is $a - ak(t)$ as $t \to \infty$ (where $k(t)$ is the Kalman gain)? .... (2p)