KTH Matematik

Solution to Homework 2
Mathematical Systems Theory, SF2832
Fall 2017
You may use min(5,(your score)/4) as bonus credit on the exam.

1. Consider the pair (c, A), where

$$A = \begin{bmatrix} -1 & a \\ -2 & -3 \end{bmatrix}$$
$$c = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

where a is a real constant.

(a) Solve the Lyapunov equation $A^T P + PA + c^T c = 0$. .... (2p)

Answer: $P = \frac{1}{4(3+2a)} \begin{bmatrix} 6+a & 1.5a \\ 1.5a & 0.5a^2 \end{bmatrix}$.

(b) Determine for what a the solution P is positive definite and positive semi-definite respectively. .... (2p)

Answer: P is positive definite if $a > -1.5$ and $a \neq 0$, positive semi-definite if $a \geq -1.5$.

2. You will in this problem derive and investigate a number of realizations for the transfer function

$$R(s) = \begin{bmatrix} \dfrac{s+2}{s+1} \\ \\ \dfrac{1}{2s+4} \end{bmatrix}$$

Answer: omitted.

(a) Determine the standard reachable realization of $R(s)$. .... (1p)

(b) Is the realization in (a) observable? .... (1p)

(c) Determine the standard observable realization of $R(s)$. .... (2p)

3. Two state space representations (A, B, C) and $\bar{A}, \bar{B}, \bar{C})$ are said to be equivalent if there exists a nonsingular matrix T such that

$$(\bar{A}, \bar{B}, \bar{C}) = (TAT^{-1}, TB, CT^{-1})$$

Use whatever method you can to verify if the following pairs of realizations are equivalent or not:

(a)

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix} \right)$$
$$(\bar{A}, \bar{B}, \bar{C}) = \left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 0.5 \end{bmatrix} \right)$$

.... (2p)

Answer: Yes, since $T = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$.

(b)

$$(A, B, C) = \left(\begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \end{bmatrix} \right)$$
$$(\bar{A}, \bar{B}, \bar{C}) = \left(\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \end{bmatrix} \right)$$

.... (2p)

Answer: No, since they do not give the same transfer function.

4. Consider a minimal SISO system

$$\dot{x} = Ax + bu$$
$$y = cx, \ x \in R^n.$$

Find further conditions on matrices A, b, c such that for any $u = kx$, the pair $(c, A + bk)$ is always observable. .... (3p)

Answer: Since (A, b) is controllable, poles of the closed-loop system can be placed arbitrarily. Thus, zeros can always be canceled, which would lead to unobservability since feedback control does not change controllability. Thus, the only way to assure observability under any feedback control is that the numerator polynomial of the transfer function must be a constant (no zero to be canceled). Namely $\tilde{c} = [c_0 \ 0 \ \cdots \ 0]$ when (A, b) is in the controllable canonical form. Then by equation (51) in the compendium we have $c[b, Ab, \cdots, A^{n-1}b] = [0, \cdots, 0, c_0]$, where $c_0 \neq 0$.

5. Suppose the following is a realization of a given $R(s)$:

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \right)$$

(a) Find a feedback control $u = Kx$ that assigns poles to $\{-1, -2, -3, -4\}$. . (2p)
Answer: There are many ways to find such a control, so it is omitted.

(b) Is the realization minimal? If not, use Kalman decomposition to find a minimal realization. .... (3p)
Answer: No. A minimal realization has dimension 2, but such a realization is not unique as we all understood.