KTH Matematik

Homework 3
Mathematical Systems Theory, SF2832
Fall 2018
You may use min(5,(your score)/4) as bonus credit on the exam.

1. Consider a state space realization (A, b, c) as follows

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} c_1 & c_2 & 1 \end{bmatrix} x,$$

where $c_1,\ c_2$ are constants.

(*a*) For what $c_1,\ c_2$ is the system observable? .... (1p)

(*b*) Design a feedback controller $u = kx$ such that dimension of the unobservable subspace for $(c, A + bk)$ is maximized while $A + bk$ is guaranteed to have one eigenvalue at -1. .... (1p)

(*c*) Design a feedback control $u = kx$ that makes $y(t) = 0,\ \forall t \geq 0$ if initially $y(0) = 0$. For the closed-loop system, discuss conditions on c_1, c_2 such that $\lim_{t\to\infty} x(t) = 0$ when $y(t) = 0,\ \forall t \geq 0$. .... (3p)

2. Consider a state space system as follows

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \alpha x_1 + u$$
$$y = x_2,$$

where α is a constant. Let $P(t, t_1)$ where $0 \leq t \leq t_1$ denote the solution to the dynamical Riccati equation associated with the following optimal control problem

$$min\ J = \int_0^{t_1} (y^2 + u^2) dt.$$

(*a*) For what α is $P(t, t_1)$ positive definite $\forall t < t_1$? .... (2p)

(*b*) For $\alpha = 0$ compute $\lim_{t_1\to\infty} P(0, t_1)$ and give the reason why the limit exists but is not positive definite. .... (3p)

3. Consider the Dynamical Riccati equation

$$\dot{P} = -A^T P - PA + PBB^T P - Q$$
$$P|_{t=t_1} = S,$$

where $Q \geq 0,\ S \geq 0$ and we use $P(t, t_1, S)$ to denote the unique solution (which implies that $P(t_1, t_1, S) = S$).

(*a*) Show that $P(t_0, t_1, 0) \leq P(t_0, \bar{t}_1, 0)$ for any $t_0 \leq t_1 \leq \bar{t}_1$ (we say $P_1 \leq P_2$ if $P_2 - P_1 \geq 0$). .... (3p)

(*b*) Assume that (A, B) is controllable. Show that $P_a = P(t, t_1, P_a)$ for any $t \leq t_1$, where $P_a = \lim_{t_1 \to \infty} P(0, t_1, 0)$. .... (2p)

4. Consider a one-dimensional system

$$\begin{aligned} x(t+1) &= ax(t) + v(t) \\ y(t) &= x(t) + w(t), \end{aligned}$$

where $a \neq 0$, v, w are uncorrelated white noises, with covariances σ, r respectively.

(*a*) Design a Kalman filter $\hat{x}(t)$ for $x(t)$. .... (1p)

(*b*) Express the covariance matrix $p(t) = E\{(x(t) - \hat{x}(t))^2\}$ in terms of a, σ, r. (2p)

(*c*) What is $a - ak(t)$ as $t \to \infty$ (where $k(t)$ is the Kalman gain)? .... (2p)