Introduction to Mathematical Systems Theory

1. Course information
2. What is mathematical systems theory?
3. Course topics

Course Information

- This is a course on linear systems
- Gives preparation for
  - SF2842 Geometric systems theory
  - SF2852 Optimal control theory
  - Advanced topics in control theory and signal processing
  - Advanced control tasks in industry (ABB, Saab, Volvo, De Laval, Process industries, Ericsson, Finance, e.t.c.)

Teachers

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Course Material

- An Introduction to Mathematical Systems Theory, Lindquist & Sand.
- Exercises in Mathematical Systems Theory, Enqvist.
- Complementary material will be handed out. It will also be posted on the course homepage.

Homework

Three optional homework sets

W1 Chapter 1 - chapter 3. Deadline on Tuesday September 23.

W2 Chapter 4 - chapter 5. Deadline on Tuesday October 7.

W3 Chapter 6 - chapter 9. Deadline is on Friday October 17.

- Each homework has five problems.
- Grading: Each homework may give maximum 3 bonus credits for the exam.

Computer Exercise

There is one optional computer exercise, where you apply the theory of the course using standard linear algebra routines in the "Control System Toolbox" in MATLAB.

- Cooperation in groups of not more than two students is allowed.
- One lab report for each group. Possibly *oral exam*.
- A successfully completed computer exercise gives you three bonus points on the final exam.
- Deadline is on October 14.

Theory Project

There is one optional theory project, where the purpose is to use what you have learnt in the course in new problems.

- Cooperation in groups of not more than two students is allowed.
- One lab report for each group. Possibly *oral exam*.
- A successfully completed theory project gives you three bonus points on the final exam.
- Deadline is on October 14.

The following topics are available

- Linear matrix equations.
- Popov-Belevic-Hautus test and block diagram algebra.

Final Written Exam

The final exam takes place on October 24, 2008 at 08:00-13:00 in room D41, V01 and V12.

Grading

Grade	A	B	C	D	E	FX
Total credit (points)	>90	76-90	61-75	50-60	45-49	41-44

- Total credit = exam score + homework score + computer exercise + theory project.
- The maximum exam score = 100. Maximum bonus from HW+CE+TP=15
- Open book exam. You may bring the lecture notes by Lindquist and Sand, β Mathematics Handbook but nothing else.

What is Mathematical Systems Theory?

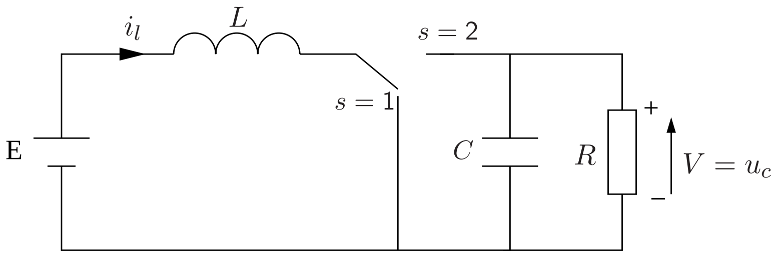
1. A mathematical theory providing the foundation for modeling, analysis, and control of dynamical systems.
2. Mathematical aspects of control theory and signal processing
3. Makes heavy use of several subfields of mathematics
   - Linear algebra and matrix theory
   - Differential equations
   - Optimization theory
   - Stochastic processes
   - Differential geometry
   - Operator theory
   - ⋮

Application Areas

- Aerospace systems
- Robotics and autonomous vehicles
- Process control
- Estimation theory (Kalman filter)
- Biology and medicin
- ⋮

Next some academic examples from the course

DC/DC Boost Converter

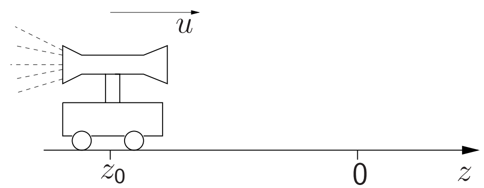


- The purpose is to amplify the voltage level.
- Periodic switching between the two switch positions.

$$s(t) = \begin{cases} 1, & t \in [kT, (k+d)T) \\ 2, & t \in [(k+d)T, (k+1)T] \end{cases}$$

where $d \in (0, 1)$.

The Rocket Car



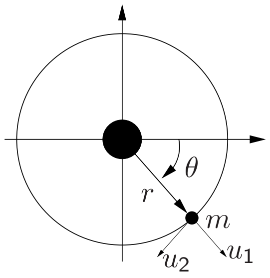
Drive the rocket car from rest at position z_0 to rest at position 0.

- There are many ways to do this.
- To obtain a unique solution we may pick the energy optimal solution.

$$\min \int_0^T u(t)^2 dt \quad \text{subj to} \quad \begin{cases} m\ddot{z} = u, \\ z(0) = z_0, \; \dot{z}(0) = 0 \\ z(T) = 0, \; \dot{z}(T) = 0 \end{cases}$$

- This is a linear quadratic control problem.

Satellite Control



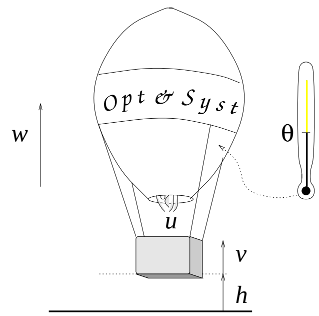
$$\ddot{r}(t) = r(t)\dot{\theta}^2(t) - \frac{k}{r(t)^2} + u_1(t)$$

$$\ddot{\theta}(t) = -\frac{2\dot{\theta}(t)\dot{r}(t)}{r(t)} + \frac{1}{r(t)}u_2(t)$$

- Can the satellite be controlled in a circular orbit? What happens if we either loose the tangential or the radial thrusts?
- What variables must be measured to control the satellite?

The Hot Air Balloon

$$\begin{cases} \dot{\theta} = -\frac{1}{\alpha}\theta + u, \\ \dot{v} = -\frac{1}{\beta}v + \sigma\theta + \frac{1}{\beta}w, \\ \dot{h} = v, \end{cases}$$



where

θ = temperature,

u = heating,

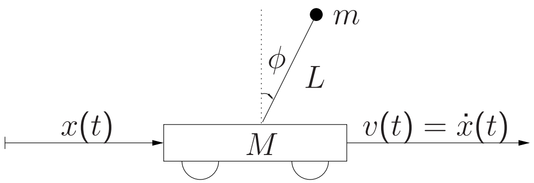
v = vertical velocity,

h = height,

w = vertical wind velocity.

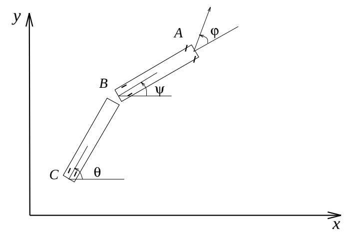
Is it possible to reconstruct θ and w from observations of height h?

The cart and pendulum



Balance the pendulum by controlling the acceleration of the cart.

Backing up a truck with a trailer along a straight line



$$\begin{bmatrix} \dot{x}_B \\ \dot{y}_B \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\varphi+\psi) \\ \sin(\varphi+\psi) \\ \sin(\varphi) \\ \cos(\varphi)\sin(\psi-\theta) \end{bmatrix}$$

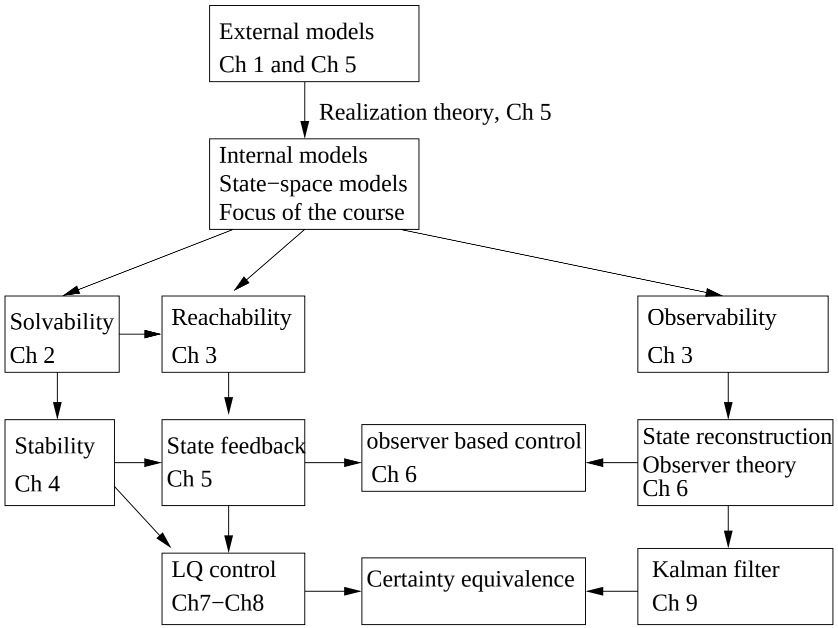
- Apply linear state feedback.

Course Topics

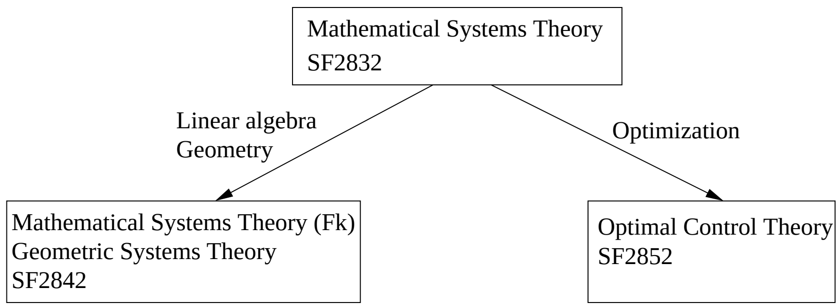
- Modeling
  - Modeling from basic principles and linearization
  - Realization theory
- Analysis
  - Reachability and observability
  - Stability
- Design
  - Design of state feedback (multi-variable and optimal)
  - Design of observer (multi-variable and optimal)
  - Observer based control (certainty equivalence)

- The focus of this course is on the state-space methods
  - Tractable for computations
  - Allows for efficient implementation
- Time-varying and multivariable systems
- Optimization aspects
- The mathematical aspects of the problems are investigated

Relation Between Course Topics



Continued Courses



- SF2812 Tillämpad matematisk programming - linjära problem
- SF2822 Tillämpad matematisk programming - ickelinjära problem