1. Consider the system

\[
\begin{align*}
\dot{x} &= \begin{pmatrix} -1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 2 \\ -1 & -1 & 2 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} -1 & -1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} u \\
y &= \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix} x.
\end{align*}
\]

(a) What is the zero dynamics? [1p]
\[\dot{z} = -z.\]
(b) Use the Rosenbrock matrix to verify your computation of the transmission zero from (a). [1p]
(c) Solve the noninteracting control problem. [1p]

Since the system has rel. degree (2, 1), the noninteracting problem is solvable. Let \( z = x_4, \xi_1 = x_1 + x_2, \xi_2 = -x_2, \xi_3 = 2x_3. \)

2. Consider the system

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_1 - 2x_2 + w_1 \\
\dot{w}_1 &= w_1 + w_2 \\
\dot{w}_2 &= w_2 \\
y &= x_1 - x_2
\end{align*}
\]

(a) Compute the invariant subspace \( x = \Pi w \) if it exists (the Matlab command \texttt{"lyap"} can be used). [2p]
(b) Is the above system (consisting of \( x \) and \( w \)) observable or not? Why so? [1p]

Not observable, since \( s=1 \) is a transmission zero.
3. Consider the car steering example:

\[
\begin{align*}
\dot{\alpha}_f &= -2\alpha_f + r + \dot{\delta}_f \\
\dot{\psi} &= r \\
\dot{r} &= -0.6\alpha_f - 2\psi + 3\delta_f + d(t),
\end{align*}
\]

where the driver’s goal is to keep the orientation straight, i.e., \(\delta_f = -0.5\psi\), and \(d(t)\) is a sinusoidal disturbance \(a\sin(2t + \theta)\) with unknown amplitude and phase.

Design an output that is a linear combination of \(\psi\) and \(r\), such that the output optimally reconstructs the disturbance in stationarity. You may use Matlab for computation. [3p]

4. Consider:

\[
\begin{align*}
\dot{x}_1 &= \alpha x_1 - x_4 + w_3 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= u_2 \\
\dot{x}_4 &= -x_3 - x_4 - u_1 + u_2 \\
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= -w_1 \\
\dot{w}_3 &= w_1 \\
e_1 &= x_2 - w_2 \\
e_2 &= x_4 - w_1
\end{align*}
\]

(a) For \(\alpha = 1\), find a control \(u = Kx + Ew\) that solves the full information output regulation problem. [2p]

1. Solve the Sylvester equation: \(x_2 = w_2, x_4 = w_1, x_3 = -w_1, x_1 = \pi_1 w, u_2 = c_2 w = -w_2, u_1 = c_1 w = u_2 - x_3 - x_4 = -w_2\).

2. Let \(f_2 x = -k_1 x_2 - k_2 x_3, f_1 x = -(p_1 + p_2 + 1) x_1 + p_2 x_4\), where all coefficients are positive.

(b) What is the real value(s) of \(\alpha\) such that the regulation problem may not be solvable? [1p]

\(\alpha = 0\).