1. Consider the system

\[ \dot{x} = g_1 u_1 + g_2 u_2, \]

where

\[
g_1 = \begin{pmatrix} 
\cos(x_3 + x_4) \\
\sin(x_3 + x_4) \\
\sin(x_4) \\
0 
\end{pmatrix}, \quad g_2 = \begin{pmatrix} 
0 \\
0 \\
0 \\
1 
\end{pmatrix}.
\]

One can view this as a more complex vehicle steering system. Define:

\[
Drive = g_1, \quad Steer = g_2, \quad Wriggle = [Steer, Drive], \quad Slide = \begin{pmatrix} 
-\sin(x_3) \\
\cos(x_3) \\
0 \\
0 
\end{pmatrix},
\]

where \([\cdot, \cdot]\) is the Lie Bracket.

- What is \([Steer, Wriggle]\) and \([Wriggle, Drive]\)? [1p]
- Is the distribution \(\text{span}\{g_1, g_2\}\) involutive? [1p]
- Show that the system is locally strongly accessible and controllable. [1p]

**Solution:** omitted.

2. Determine if each of the following statements is true or false.

- Consider the consensus control problem in \(R^2\) with four agents: \(x_i = u_i, \ x_i \in R^2, \ u_i \in R^2\). If the initial positions of the four agents form a rectangle (with the position of each agent as a vertex), then as the consensus control is applied, no agent can ever move outside the rectangle. [2p]

  **Solution:** True since all agents can only move inside the convex hull of the initial conditions.

- If a nonlinear control system is exactly linearizable as defined in the compendium, then it is exponentially stabilizable by feedback control. [2p]

  **Solution:** True since the exactly linearized system is controllable by definition.
3. Consider
\[
\begin{align*}
\dot{x}_1 &= \alpha x_1 + x_1^5 - x_1^4 x_2 \\
\dot{x}_2 &= x_1 - x_2 - \beta x_1^3,
\end{align*}
\]
where $\alpha$ and $\beta$ are constant.

- Discuss for what value of $\alpha$ the stability of the origin does not depend on $\beta$. [1p]
  \textbf{Solution:} When $\alpha \neq 0$, by the principle of stability in the first approximation.
- For the remaining case analyze the stability in terms of $\beta$. [2p]
  \textbf{Solution:} When $\alpha = 0$, using the center manifold theory we have that the system is asymptotically stable if $\beta > 0$, unstable if $\beta < 0$, and critically stable if $\beta = 0$.

4. Consider in a neighborhood $N$ of the origin
\[
\begin{align*}
\dot{x}_1 &= -x_2 - x_1^3 \\
\dot{x}_2 &= x_1 + x_2^2 + u \\
\dot{x}_3 &= x_1 - x_3^3 \\
y &= x_1.
\end{align*}
\]
- Convert the system locally into the normal form. [2p]
  \textbf{Solution:} Let $\xi_1 = x_1$, $\xi_2 = x_2$, $z = x_3$. The rest is omitted.
- Can the high gain output control $u = -ky$, $k > 0$ stabilize the system locally? [1p]
  \textbf{Solution:} No.
- Is the system exactly linearizable (without considering the output) around the origin? [2p]
  \textbf{Solution:} Yes if for example we choose $\lambda(x) = x_3$. 