1. (a) \[ \text{Wriggle} = [\text{Steer, Drive}] = \begin{pmatrix} \sin(x_3 + x_4) \\ -\cos(x_3 + x_4) \\ -\cos(x_4) \\ 0 \end{pmatrix}, \text{ and} \]
\[ [\text{Steer, Wriggle}] = \begin{pmatrix} -\cos(x_3 + x_4) \\ -\sin(x_3 + x_4) \\ -\sin(x_4) \\ 0 \end{pmatrix} = -g_1, \]
\[ [\text{Wriggle, Drive}] = \begin{pmatrix} -\sin(x_3) \\ \cos(x_3) \\ 0 \\ 0 \end{pmatrix} = \text{Slide}. \]

(b) No, since \([g_2, g_1] = \text{Wriggle} \notin \text{span}\{g_1, g_2\} \).

(c) \(R_2(x) = \text{span}\{g_1, g_2, \text{Wriggle, Slide}\} \), and the determinant of the matrix \((g_2 \quad g_1 \quad \text{Wriggle} \quad \text{Slide})\) is -1, which means \(R_c(x) = R_2(x)\) and \(\dim R_c(x) = 4\). So the system is locally strongly accessible. Because the \(f\) function is zero in this system, it is also controllable.

2. (a) We just need the average of the initial positions to be \((0, 0)\), which leads to the condition
\[ x_3^{(1)} + x_4^{(1)} = 2, \text{ and } x_3^{(2)} + x_4^{(2)} = 0. \]

(b) The system
\[ \dot{x} = \begin{pmatrix} -2k & k & 0 & k \\ k & -2k & k & 0 \\ 0 & k & -2k & k \\ k & 0 & k & -2k \end{pmatrix} x \]
with \(k \geq 1.5\) fulfills all the conditions.

3. (a) When \(\alpha < -1\), the system is asymptotically stable. When \(\alpha > -1\), the system is unstable.

(b) When \(\alpha = -1\), the system can be written as
\[ \dot{w} = -w^3 + O(w^4) \]
on the center manifold, which is asymptotically stable. Therefore the original system is asymptotically stable when $\alpha = -1$.

4. (a) The relative degree of the system is 2. Let $\xi_1 = y = x_1 + x_2$, $\xi_2 = \dot{\xi}_1 = x_1 + x_2 + x_3$ and $z = 2x_1 - x_3$. The normal form of the system will be

$$
\begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \xi_1 + \left(\frac{3}{2} \xi_1 - \frac{1}{2} \xi_2 + z\right)^3 + 2u \\
\dot{z} &= -z + 2\xi_1 + 3\left(\frac{3}{2} \xi_1 - \frac{1}{2} \xi_2 + z\right)^3
\end{align*}
$$

(b) The zero dynamics is $\dot{z} = -z + 3z^3$, which is a stable system. Therefore, the system can be locally stabilized around the origin.

(c) $ad_{fg} = [f, g] = \begin{pmatrix} 1 + 3x_2^2 \\ -3 - 3x_2^2 \\ 2 - 3x_2^2 \end{pmatrix}$ and $ad_{ad_{fg}}^2 g = [f, ad_{fg}] = \begin{pmatrix} -6x_2 \\ 6x_2 \\ 6x_2 \end{pmatrix} \notin \text{span}\{g, ad_{fg}\}$, which implies $\text{span}\{g, ad_{fg}\}$ is not involutive, and the system cannot be exactly linearized around the origin.