KTH Matematik

SF2842: Geometric Control Theory

Homework 1

Due November 19, 16:50, 2007

You may discuss the problems in group (maximal **two** students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. [4p]. Consider the system

$$\begin{aligned}\dot{x} &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 \end{pmatrix} x + \begin{pmatrix} 3 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} u \\ y &= (0\ 0\ 1\ 0)x,\end{aligned}$$

where $x = (x_1, x_2, x_3, x_4)^T$.

(a) Is the system controllable?

(b) Is the system observable?

(c) Compute $\mathcal{V}^*$ and $\mathcal{R}^*$ contained in $\mathcal{V}^*$, and find all friends F of $\mathcal{V}^*$.

2. [4p]. Consider the same system as in Problem 1. Suppose we are given a three dimensional space XYZ. We identify x_1 in the system as x, x_2 as y, x_3 as z and x_4 as $\dot{z}$.

(a) Consider two points on the XY plane: $(0, -5)$ and $(3, 4)$. Show that these two points can be connected by the trajectory that consists of the two line segments connecting the respective point to the origin.

(b) Suppose the system starts at $(0, -5)$ and reaches $(3, 4)$ in 10 units of time at constant speed, design an open-loop controller for that.

3. [5p]. Consider

$$\begin{aligned}\dot{x} &= Ax + Bu + Ew \\ y &= Cx,\end{aligned}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad E = \begin{pmatrix} 1 \\ -a \\ 1 \end{pmatrix}, \quad C = (a\ 1\ 0).$$

(a) Show DDP is solvable for both the cases $a = 1$ and $a = -1$.

(b) For $a = 1$, Find a $u = Fx$ that solves the DDP problem while makes the closed-loop system stable, i.e. $A + BF$ has only eigenvalues with negative real part. Can we find such an F for $a = -1$?

(c) Verify that (A, E) is controllable. Explain why even in this case the DDP problem could still be solvable (namely $w(t)$ can not at all control the output).

4. [3p]. Consider

$$
\begin{aligned}
\dot{x}_1 &= -2x_1 + x_4 + u_1 \\
\dot{x}_2 &= x_2 + 2u_2 \\
\dot{x}_3 &= x_2 + x_4 + u_2 \\
\dot{x}_4 &= x_3 \\
y_1 &= x_1 - x_3 \\
y_2 &= x_4
\end{aligned}
$$

(a) What is the relative degree for the system?

(b) Convert the system into the normal form and compute the zero dynamics.

(c) What is the $\mathcal{R}^*$ contained in $Ker\ C$?