KTH Matematik

SF2842: Geometric Control Theory

Homework 1

Due November 18, 16:50pm, 2009

You may discuss the problems in group (maximal **two** students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. [5p]. Consider the system

$$
\begin{aligned}
\dot{x} &= \begin{pmatrix} 0 & 2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} u \\
y &= (0\ 1\ -1\ 0)x.
\end{aligned}
$$

(a) Compute $\mathcal{V}^*$ and express all friends F of $\mathcal{V}^*$.

(b) Compute $\mathcal{R}^*$ that is contained in $ker\ C$.

(c) If we want to use $u = Fx$ to induce unobservability, what is the maximal possible dimension of the unobservable subspace for the closed-loop system?

2. [2p]. Consider

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{aligned}
$$

(a) Show the controllable subspace is A-invariant.

(b) Show the unobservable subspace is A-invariant.

3. [4p]. Consider

$$
\begin{aligned}
\dot{x} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u + \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} w, \\
y &= (1\ 1\ 0)x
\end{aligned}
$$

where w is the disturbance.

(a) Derive the minimum constraint on $d_i,\ i = 1, 2, 3$ such that DDP is solvable.

(b) Can we find a $u = Fx + v$ that solves the DDP problem in (a) while makes the closed-loop system stable, i.e. $A + BF$ has only eigenvalues with negative real part, and why?

4. [4p]. Consider

$$\begin{aligned}
\dot{x}_1 &= -x_1 + x_4 + u_1 \\
\dot{x}_2 &= -x_1 - a u_2 \\
\dot{x}_3 &= -x_2 - x_3 - 2u_1 + u_2 \\
\dot{x}_4 &= x_2 + 2u_1 - u_2 \\
y_1 &= x_1 - x_2 \\
y_2 &= x_3 + x_4,
\end{aligned}$$

where a is a constant.

(a) Discuss conditions on a such that the system has a relative degree.

(b) For $a = 0$, convert the system into the normal form and compute the zero dynamics.