KTH Matematik

SF2842: Geometric Control Theory

Homework 1

Due November 16, 16:50pm, 2011

You may discuss the problems in group (maximal **two** students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. [5p]. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ -1 & 1 \\ 1 & 1 \end{pmatrix} u \\ y &= (1\ 1\ 0\ 0)x. \end{aligned}$$

(a) Compute $\mathcal{V}^*$ and express all friends F of $\mathcal{V}^*$.

(b) Compute $\mathcal{R}^*$ that is contained in $ker\ C$.

(c) Can we find a friend F of $\mathcal{V}^*$ such that $(A + BF)$ has all eigenvalues with negative real parts?

2. [6p]. Consider

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned}$$

where $x \in R^n$, $u \in R^m$ and $y \in R^p$.

(a) Show the controllable subspace is (A+BF)-invariant for any F.

(b) Assume further that $CA^kB \neq 0$, for some $k < n$. Show the unobservable subspace is not (A+BF)-invariant for all F.

(c) Suppose (C, A) is observable and the dimension of $\mathcal{V}^*$ is greater or equal to one. Show it is not possible to express a friend F of $\mathcal{V}^*$ as $F = LC$, namely it is not possible to use output feedback to make $\mathcal{V}^*$ invariant.

3. [4p]. Consider

$$\begin{aligned} \dot{x}_1 &= x_2 + x_3 + x_4 \\ \dot{x}_2 &= -x_1 - \alpha u \\ \dot{x}_3 &= -x_2 - x_3 + u \\ \dot{x}_4 &= x_2 - u \\ y &= x_3 + x_4, \end{aligned}$$

where α is a constant.

(a) Convert the system into the normal form and compute the zero dynamics.

(b) Computer $\mathcal{V}^*$ and $\mathcal{R}^*$ in $ker\ C$.

(c) For what α we can find a friend f of $\mathcal{V}^*$ such that $(A + bf)$ is a stable matrix?