KTH Matematik

SF2842: Geometric Control Theory

Homework 1

Due February 10, 16:59, 2015

You may discuss the problems in group (maximal **two** students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. Consider the system

$$\begin{aligned}\dot{x} &= Ax + Bu = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} u \\ y &= Cx = (1\ 0\ 0\ 0)x. \end{aligned} \tag{1}$$

(a) Is the system controllable? .... (1p)

(b) Let $x(t, u)$ denote the solution to (1) with control $u(t)$ and initial condition $x(0, u) = x_0$. Compute the subspace S of initial conditions x_0 that make $x(t, u) \in Ker\ C\ \forall t \geq 0$ for some $u(t)$, and design such a $u(t)$ as feedback control. .... (3p)

(c) For any $x_0 \in S$, where S is the subspace you computed in (b), and any $t_1 > 0$, can we always find a $u(t)$ such that $x(t_1, u) = 0$? .... (1p)

(d) For any $x_0 \in S$, where S is the subspace you computed in (b), and any $t_1 > 0$, can we always find a $u(t)$ such that $x(t_1, u) = 0$, and $x(t, u) \in S,\ 0 \leq t \leq t_1$? (2p)

2. Consider an observable system

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx,\end{aligned}$$

where $x \in R^n,\ u \in R^1,\ y \in R^1$.

(a) Show the controllable subspace $\mathcal{R}$ is $(A + BF)$-invariant for any F. .... (1p)

(b) List all controllability subspaces. .... (1p)

(c) Show that $(C,\ A+BF)$ is also observable for almost all F, namely the elements of those F that make $(C,\ A + BF)$ not observable can be defined by a set of algebraic constraints. .... (3p)

3. Consider

$$\begin{aligned}\dot{x} &= Ax + Bu + Ew \\ y &= Cx,\end{aligned}$$

where

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & a & 0 \\ 2 & 0 & 1 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},\ C = (1\ 0\ 0),\ E = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix},$$

where a and d_1, d_2, d_3 are constants.

(a) For what E is DDP solvable? .... (2p)

(b) For what a can we find a $u = Fx$ that solves the DDP problem while makes the closed-loop system stable? i.e. $A + BF$ has only eigenvalues with negative real part. .... (2p)

(c) What is R^*? .... (1p)

4. Consider

$$\begin{aligned}\dot{x}_1 &= x_1 + x_3 + u_1 \\ \dot{x}_2 &= -x_1 + x_3 - u_1 \\ \dot{x}_3 &= x_2 - x_3 + x_4 + u_2 \\ \dot{x}_4 &= 2x_1 + x_4 + u_1 \\ y_1 &= x_1 + x_2 \\ y_2 &= x_4\end{aligned}$$

(a) What is the relative degree for the system? .... (1p)

(b) Convert the system into the normal form and compute the zero dynamics.(3p)

(c) When $y(t) = 0\ \forall t \geq 0$, what happens to $x(t)$ as $t \to \infty$? .... (1p)