KTH Matematik

SF2842: Geometric Control Theory

Homework 2

Due December 5, 16:50pm, 2011

You may discuss the problems in group (maximal two students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. Consider the system

$$\begin{aligned}\dot{x} &= \begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 1 & 1 \end{pmatrix} x,\end{aligned}$$

where a is a constant.

(a) For what a does the system have a relative degree? [1p]

(b) When the system has a relative degree, compute the zero dynamics. [2p]

(c) Use the Rosenbrock matrix to verify your computation of the transmission zeros from (b). [2p]

2. Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -x_1 - 3x_2 - 3x_3 + u \\ \dot{w}_1 &= w_2 \\ \dot{w}_2 &= -w_1 \\ u &= w_1 \\ y &= c_1 x_1 + c_2 x_2 + x_3,\end{aligned}$$

where $c_1,\ c_2$ are constant and $c_1 - c_2 + 1 \neq 0$.

(a) Compute the invariant subspace $x = \Pi w$. [2p]

(b) For what value(s) of c_1, c_2 is the above system (consisting of x and w) possibly unobservable? Explain why. [1p]

(c) Design c_1, c_2 such that $y(t) = u(t)$ in the steady state. [2p]

3. Consider:

$$\begin{aligned}
\dot{x}_1 &= x_2 + x_4 \\
\dot{x}_2 &= x_2 + u_1 \\
\dot{x}_3 &= -2x_3 + w_3 + u_2 \\
\dot{x}_4 &= x_1 - \alpha x_3 - x_4 + u_2 \\
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= -w_1 \\
\dot{w}_3 &= 0 \\
e_1 &= x_1 - 2w_1 \\
e_2 &= x_4 - 3w_2
\end{aligned}$$

(a) For what α is the full information output regulation problem solvable? [2p]

(b) For what α is the error feedback output regulation solvable? [1p]

(c) For $\alpha = 1$, solve the the full information output regulation problem. [2p]