1. Consider the system

\[
\begin{align*}
\dot{x} &= \begin{pmatrix} 1 & a & 0 & 0 \\ 0 & 2 & a & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} u \\
y &= \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} x,
\end{align*}
\]

where \( a \) is a constant.

(a) For what \( a \) does the system have a relative degree? [1p]

(b) When the system has a relative degree, compute the zero dynamics. [2p]

(c) Use the Rosenbrock matrix to verify your computation of the transmission zeros for \( a = -2 \). [2p]

2. Consider the system

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_1 - 2x_2 + w_1 \\
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= 2w_2 \\
y &= ax_1 + x_2,
\end{align*}
\]

where \( a \neq 0 \) is constant.

(a) Compute the invariant subspace \( x = \Pi w \). [2p]

(b) For what value(s) of \( a \) is the above system (consisting of \( x \) and \( w \)) unobservable? Explain why. [2p]

3. Consider the car steering example:

\[
\begin{align*}
\dot{\alpha}_f &= -2\alpha_f + r + \delta_f \\
\dot{\psi} &= r \\
\dot{r} &= -0.6\alpha_f - 2\psi + 3\delta_f + d(t),
\end{align*}
\]
where the driver’s goal is to keep the orientation straight ($\delta_f = -0.5\psi$), $d(t)$ is a sinusoidal disturbance $a \sin(2t + \theta)$ with unknown amplitude and phase.

Design an output (you may use Matlab) that is a linear combination of $\psi$ and $r$, such that the output optimally reconstructs the disturbance in stationarity and use Matlab simulation plot to verify your design. [4p]

4. Consider:

$$
\begin{align*}
\dot{x}_1 &= 2x_2 + x_4 \\
\dot{x}_2 &= x_2 + x_3 + u_2 \\
\dot{x}_3 &= x_3 + w_3 + u_1 \\
\dot{x}_4 &= -\alpha x_3 - x_4 + u_1 \\
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= -w_1 \\
\dot{w}_3 &= 0 \\
e_1 &= x_1 - w_1 \\
e_2 &= x_4 - w_2 \\
\end{align*}
$$

(a) For what $\alpha$ is the full information output regulation problem solvable? [2p]

(b) For $\alpha = 0$, solve the the full information output regulation problem. [2p]