1. Consider the system

\[ \dot{x} = g_1 u_1 + g_2 u_2, \]

where

\[ g_1 = \begin{pmatrix}
\cos(x_3 + x_4) \\
\sin(x_3 + x_4) \\
\sin(x_4) \\
0
\end{pmatrix}, \quad g_2 = \begin{pmatrix}
0 \\
0 \\
0 \\
1
\end{pmatrix}. \]

One can view this as a more complex vehicle steering system. Define:

- \( Drive = g_1, \) \( Steer = g_2, \) \( Wriggle = [Steer, Drive], \) \( Slide = \begin{pmatrix}
-\sin(x_3) \\
\cos(x_3) \\
0 \\
0
\end{pmatrix}, \)

where \([,]\) is the Lie Bracket.

- What is \([Steer, Wriggle]\) and \([Wriggle, Drive]\)? [1p]
- Is the distribution \( \text{span}\{g_1, g_2\} \) involutive? [1p]
- Show that the system is locally strongly accessible and controllable. [1p]

2. Determine and justify if each of the following statements is true or false.

- Consider the consensus control problem in \( R^3 \) with \( N \) agents: \( \dot{x}_i = u_i, \) \( x_i \in R^3, \) \( u_i \in R^3, \) \( i = 1, \ldots, N. \) If the initial positions of the agents are contained in a closed sphere \( (\|x - x_0\|^2 \leq r^2) \), then for any connection graph, as the consensus control \( u_i = \sum_{j \in N_i} (x_j - x_i) \) is applied, no agent can ever move outside the sphere. [2p]
- Consider a smooth nonlinear control system

\[ \dot{x} = f(x) + g(x)u, \]

where \( f(0) = 0. \) If it is not controllable around the origin, then \( x = 0 \) is not stabilizable by any differentiable feedback control, assuming \( x = 0 \) is unstable under \( \dot{x} = f(x). \) [1p]
3. Consider
\[
\begin{align*}
\dot{x}_1 &= \alpha x_1 + x_1^5 - 2x_1^4 x_2 \\
\dot{x}_2 &= x_1 - 2x_2 + \beta x_1^3,
\end{align*}
\]
where \(\alpha\) and \(\beta\) are constant.

- Discuss for what values of \(\alpha\) the stability of the origin does not depend on \(\beta\). [1p]
- For the remaining case analyze the stability in terms of \(\beta\). [3p]

4. Consider in a neighborhood \(N\) of the origin
\[
\begin{align*}
\dot{x}_1 &= x_2 - x_1^3 \\
\dot{x}_2 &= -x_2 + x_3^2 + u \\
\dot{x}_3 &= x_1^3 + x_3 u \\
y &= x_1.
\end{align*}
\]

- Convert the system locally into the normal form. [3p]
- Can the system be stabilized locally around the origin? [1p]
- Is the system exactly linearizable (without considering the output) around the origin? [2p]