KTH Matematik

SF2842: Geometric Control Theory

Homework 3

Due March 11, 16:50pm, 2015

You may discuss the problems in group (maximal two students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2^2 \\ \dot{x}_2 &= u,\end{aligned}$$

a. Show that the system is locally strongly accessible. [2p]

b. Show that the system is not controllable. (Hint: try to find some points that are not reachable from a given point) [2p]

2. Determine and justify if each of the following statements is true or false.

- Consider the consensus control problem in R^2 with N agents: $\dot{x}_i = Ax_i + Bu_i,\ x_i \in R^2,\ u_i \in R,\ i = 1, \cdots, N$, (A, B) is controllable and the neighbor graph is connected. If the initial positions of the agents are contained in a disc, then as a consensus control (i.e. a control that assures consensus reaching as $t \to \infty$) $u_i = K\sum_{j\in N_i}(x_j - x_i)$ is applied, no agent can move outside the disc at any time. [2p]
- Consider a smooth nonlinear control system defined in a neighborhood N of the origin
$$\dot{x} = f(x) + g(x)u,$$
where $f(0) = 0$, and $x = 0$ of $\dot{x} = f(x)$ is unstable. If the system is not controllable, then $x = 0$ is not asymptotically stabilizable by any feedback control. [2p]

3. Consider

$$\begin{aligned}\dot{x}_1 &= \alpha x_1 + x_2 + x_1^3 + 2x_1^2 x_2 \\ \dot{x}_2 &= -x_1^3 - x_2,\end{aligned}$$

where α is constant.

Determine the stability of the origin by dividing α into different ranges so that you use respectively

a. The principle of stability in first approximation to analyze the stability; [1p]

b. The center manifold theory to analyze the stability. [2p]

4. Consider in a neighborhood N of the origin

$$\begin{aligned} \dot{x}_1 &= x_1^3 + x_2 + u \\ \dot{x}_2 &= x_1 - x_2^3 + x_3 - u \\ \dot{x}_3 &= 2x_2^3 - 2x_3 + 2u \\ y &= 2x_2 + x_3. \end{aligned}$$

a. Convert the system locally into the normal form. [2p]

b. What is the zero dynamics? [1p]

c. Is the system exactly linearizable (without considering the output) around the origin? [2p]