Homework 3, SF2842

Sketch of Solutions

1. Omitted.

2.

2.1. True. Since the convex hull of the N points in R^2 lies inside the disc in this case, and $\dot{x}_i$ for each agent by definition points inwards the convex hull.

2.2. True. Since by the definition of ~~[illegible]~~ exact linearization, the linearized system, which is "equivalent" to the original system, is controllable.

3.

3.1. $\alpha \neq 0$. We can apply stability in the first approximation in this case.

3.2. When $\alpha = 0$, we apply center manifold theory. On the center manifold, we have (when $\beta \neq 0$)

$$\dot{w} = \beta w^5 + o(|w|^5)$$

$\Rightarrow$ $\beta < 0$, asym. stable

$\beta > 0$, unstable.

$\beta = 0$ $\Rightarrow$ $\dot{w} = 0$, stable.

4.

4.1. It is easy to calculate that

$r = 2$.

We chose $\xi_1 = x_1$, $\xi_2 = 2x_3 - x_1^3$

and $z = e^{x_2}\cos(x_3) - 1$

and the normal form follows.

4.2. The zero dynamics is

$\dot{z} = -z^3 - z^4$

asym. stable!

$\Rightarrow$ the system is locally stabilizable.

4.3. Yes. We can, for example, chose

$\lambda(x) = e^{x_2}\cos(x_3) - 1$

to show that.