KTH Matematik

Final Exam of SF2842 Geometric Control Theory

December 19 2007, 14-19

Examiner: Xiaoming Hu, phone 790 71 80, mobile 070-796 78 31.

Allowed written material: All course material (except the old exams and their solutions) and β mathematics handbook.

Solution methods: All conclusions must be properly motivated. Note: the problems are not necessarily ordered in terms of difficulty.

Note! Your personnummer must be stated on the cover sheet. Number your pages and write your name on each sheet that you turn in!

Preliminary grades: 31 points give grade C, 37 points give grade B and 43 points guarantee grade A (maximum is 50).

---

1. Determine if each of the following statements is *true* or *false* and **motivate** (no motivation no score) your answer briefly (for example, to show a statement is false, a counter-example is enough).

(a) Consider a linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^m$.

Let $S = ker(C)$. If (C, A) is observable, then $R^* = 0$ (the maximal reachability subspace in S). .... (2p)

(b) If system (1) has relative degree $(r_1, \cdots, r_m)$, then $dim(V^*) = n - \sum_{i=1}^m r_i$ and $R^* = 0$. .... (2p)

(c) Now suppose the system is influenced by disturbance Pw (namely we add Pw to the right hand side of equation (1)). Then the solvability of DDP is a necessary condition for solving any full information output regulation problem. .... (2p)

(d) Consider a nonlinear single-input system

$$\dot{x} = f(x) + g(x)u \tag{2}$$

where $x \in R^n$, $f, g \in C^\infty$ and $f(0) = 0$. Let $A = \frac{\partial f}{\partial x}|_{x=0}$, $b = g(0)$. If (A, b) is not controllable, then the system is not exactly linearizable around the origin. (2p)

(e) We consider the same nonlinear system as in question (d). A necessary condition for the existence of an output mapping $h(x)$ such that the system has a relative degree $r \leq n$ at the origin is that the matrix A is not identically zero. ... (2p)

2. Consider the system

$$\begin{aligned}\dot{x} &= \begin{pmatrix} -1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 0 & -3 \end{pmatrix} x + \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} u \\ y &= (0\ -1\ 1\ 0)x.\end{aligned}$$

(a) Find a feedback $u = Fx$ that would maximize the unobservable subspace. What is the unobservable subspace? . . . . (6p)

(b) Find R^* contained in V^*. . . . . (2p)

(c) Can we find a control $u = Fx$, where F is a friend of V^*, such that the closed-loop system is exponentially stable? . . . . (2p)

3. Consider the system

$$\begin{aligned}\dot{x} &= \begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \\ \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} x.\end{aligned}$$

Choose two outputs out of the three (namely, disregard the third output), such that

(a) the noninteracting control problem is solvable, ... (6p)
and

(b) the zero dynamics is asymptotically stable. . . . . (4p)

4. Consider:

$$\begin{aligned}\dot{x} &= Ax + bu + Pw \\ \dot{w} &= \Gamma w \\ y &= cx,\end{aligned}$$

where

$$A = \begin{pmatrix} 0 & 9 & 0 \\ -1 & -\alpha & -1 \\ 0 & 0 & -\alpha \end{pmatrix},\ \Gamma = \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix},\ b = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix},\ P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix},\ c = \begin{pmatrix} 3 & 0 & 0 \end{pmatrix}$$

and α is a **positive constant**.

(a) Show the **existence** of an invariant subspace $x = \Pi(\alpha)w$ for the system when u is set to zero (no exact Π is needed) . . . . (2p)

(b) Find the values of α, such that the reduced system

$$\begin{aligned} \dot{w} &= \Gamma w \\ y &= c\Pi(\alpha)w \end{aligned}$$

is not observable. .... (4p)

(c) Show for almost all values of α, the full information output regulation problem with the tracking error $e = y - w_2$ is solvable (*a solution is not required*) . What are the values of α that may make the problem unsolvable?
.... (4p)

5. Consider in a neighborhood N of the origin

$$\begin{aligned} \dot{x}_1 &= \alpha \tan(x_1) + x_2 \tan(x_1) + 2x_3 + \cos(x_3)u \\ \dot{x}_2 &= -\alpha \sin^2(x_1) - \sin^3(x_1) - x_2 + x_3^3 \\ \dot{x}_3 &= x_3 - \cos(x_1)u \\ y &= x_3, \end{aligned}$$

where α is a constant.

(a) Convert the system into the normal form (*you need to specify the new coordinates explicitly, but do not need to calculate the right hand side of the normal form in every detail*). .... (4p)

(b) Analyze the stability of the zero dynamics with respect to the value of α. (4p)

(c) Consider the same nonlinear system but without the output. Show that the exact linearization problem is not solvable (*Hint: this does not have to involve a lot of calculations*). ....(2p)