

# SF2863 Systems Engineering, 7.5 HP

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Optimization and Systems Theory  
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November 5, 2013



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- 2 Examples of Applications
- 3 Introduction to Markov Chains

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- Main Literature: “Introduction to operations research”, Ninth edition, by Hillier and Lieberman.

Should be available at the KTH Bookshop

The following material is sold at the math student office, Lindstedtsv 25.

- Exercises in SF2863 Systems Engineering, 2011.

Further material will be posted on the homepage.

On the course homepage

<http://www.math.kth.se/optsys/grundutbildning/kurser/SF2863/>  
you can find

- 1 a preliminary schedule
- 2 reading instructions, recommended exercises etc.
- 3 home assignments, rules and information about deadlines
- 4 these slides

There will be two voluntary home assignments.

- HA 1: Markov chain/process example - the ferry (2 bonus points)
- HA 2: Spare parts optimization (4 bonus points)

The maximal result on the exam (not counting bonus points) is 50 points.

Preliminary grade limits:

Grade	A	B	C	D	E	FX
Points	43-50	38-42	33-37	28-32	25-27	23-24

- At the exam a brief formula sheet will be handed out. No other tools, such as calculators, are allowed.
- The first written exam is January 13, 2014, at 14.00-19.00.
- It is necessary to sign up for the exam, and it can be done on “My pages”, Nov. 25 - Dec. 8.

## Course in Systems Engineering with Introduction to Markov Chain/Process theory.

“We use statistics, probability theory and differential/difference equations to build mathematical models for processes, combine them to complex systems, analyze them and optimize to find the best control/management policy.”



- 1 Markov chains/processes
- 2 Queueing theory
- 3 Spare parts optimization
- 4 Marginal Allocation
- 5 Deterministic/Stochastic Inventory theory
- 6 Dynamic Programming
- 7 Markov Decision Processes



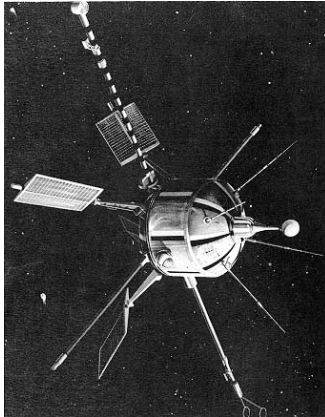
# Newsboy Problem

How many newspapers should the salesman buy each day ?



# Marginal Allocation Problem

Where should you use redundancy to get the best reliability ?  
(relative to the weight)





The diagram shows two nodes, E and A, arranged vertically. Node E is a grey circle with a white 'E' inside. Node A is a blue circle with a white 'A' inside. There are four curved orange arrows representing recurrent connections: one from E to E (top), one from E to A (middle), one from A to A (bottom), and one from A to E (middle-left).

Markov Chains with state  $X_t$  where  $t = 0, 1, 2, \dots$ .

- $X_t$  = wind condition  $\{1 = \text{Calm}, 2 = \text{breeze}, 3 = \text{storm}, \}$  at a particular place on day  $t$ .
- $X_t$  = number of items in stock of a particular item on day  $t$ .
- $X_t$  = accumulated sum of points after  $t$  rolls of a die.
- $X_t$  = number of rabbits living on Gärdet at time  $t$ .
- $X_t$  = number of complaint phone calls to the help desk at day  $t$ .
- $X_t$  = condition of patient  $\{1 = \text{stable}, 2 = \text{manic}, 3 = \text{depressive}\}$  on day  $t$ .

# Some useful results from probability theory



The **conditional probability** of  $A$  given  $B$  is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

## Law of total probability

If  $B_n$  form a partition of the sample space, i.e., the  $B_n$  are disjoint and the union is the whole sample space, then

$$Pr(A) = \sum_n Pr(A|B_n)Pr(B_n)$$

In particular, if  $X$  and  $Y$  are discrete valued stochastic variables, then

$$Pr(X = x) = \sum_y Pr(X = x|Y = y)Pr(Y = y)$$