## SF2863 Systems Engineering, 7.5 HP

Lecturer: Per Enqvist

Optimization and Systems Theory Department of Mathematics KTH Royal Institute of Technology

November 2, 2015





### Course Information





Introduction to Markov Chains



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• Main Literature: "Introduction to operations research", Tenth edition, by Hillier and Lieberman.

Available at the KTH Bookshop

The following material will be available on *Bilda* for registered students:

• Exercises in SF2863 Systems Engineering, 2014.

Further material will be posted on the homepage.



On the course homepage

http://www.math.kth.se/optsyst/grundutbildning/kurser/SF2863/ you can find

- a preliminary schedule
- reading instructions, recommended exercises etc.
- In home assignments, rules and information about deadlines
- these slides



There will be two voluntary home assignments.

- HA 1: Markov chain/process example the ferry (2 bonus points)
- HA 2: Spare parts optimization (4 bonus points)





The maximal result on the exam (not counting bonus points) is 50 points.

Preliminary grade limits:

Grade	A	В	С	D	E	FX
Points	43-50	38-42	33-37	28-32	25-27	23-24

- At the exam a brief formula sheet will be handed out. No other tools, such as calculators, are allowed.
- The first written exam is January 11, 2016, at 14.00-19.00.
- It is necessary to sign up for the exam, and it can be done on "My pages", Nov. 2 - Dec. 13.



Course in Systems Engineering with Introduction to Markov Chain/Process theory.

"We use statistics, probability theory and differential/difference equations to build mathematical models for processes, combine them to complex systems, analyze them and optimize to find the best control/management policy."

## **Course elements**

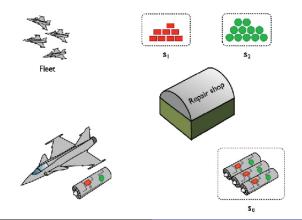


- Markov chains/processes
- Queueing theory
- Spare parts optimization
- Marginal Allocation
- Oeterministic/Stochastic Inventory theory
- Oynamic Programming
- Ø Markov Decision Processes

# **Spare Parts Optimization**



How many spareparts of each type should be held, in which location, and where should they be repaired?



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## Newsvendor Problem



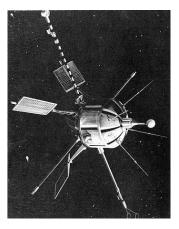
#### How many newspapers should the vendors buy each day ?



# Marginal Allocation Problem



Where should you use redundance to get the best reliability ? (relative to the weight)



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# Queueing theory



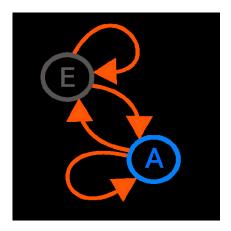


#### Figure : Queueing strategy





Example with two states, 'E' and 'A'.





Markov Chains with state  $X_t$  where  $t = 0, 1, 2, \cdots$ .

- *X<sub>t</sub>* = wind condition {1 = Calm, 2 = breeze, 3 = storm, } at a particular place on day *t*.
- $X_t$  = number of items in stock of a particular item on day t.
- $X_t$  = accumulated sum of points after *t* rolls of a die.
- $X_t$  = number of rabbits living on Gärdet at time t.
- $X_t$  = number of complaint phone calls to the help desk at day t.
- X<sub>t</sub> = condition of patient {1 = stable, 2 = manic, 3 = depressive} on day *t*.

### The **conditional probability** of *A* given *B* is defined by

$$P(A|B) = rac{P(A \cap B)}{P(B)}$$

#### Law of total probability

If  $B_n$  form a partition of the sample space, i.e., the  $B_n$  are disjoint and the union is the whole sample space, then

$$Pr(A) = \sum_{n} Pr(A|B_n)Pr(B_n)$$

In particular, if X and Y are discrete valued stochastic variables, then

$$Pr(X = x) = \sum_{y} Pr(X = x | Y = y) Pr(Y = y)$$