Derivation of mean number of customers in the queue

There was some problem with the derivation of the number of customers in queue for $M|M|s$ queues in class. The reason behind it was that I mixed up the definition of ρ. It is defined as $\rho = \lambda/\mu$ in the book for $M|M|1$ systems, and as $\rho = \lambda/(s\mu)$ in the book for $M|M|s$ systems.

The steady state probabilities are given by

$$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & \text{if } 0 \le n \le s \\ \dfrac{(\lambda/\mu)^n}{s! s^{n-s}} P_0 & \text{if } n \ge s \end{cases}$$

The correct derivation of L_q should be

$$\begin{aligned} L_q &= \sum_{n=s}^{\infty} (n-s) P_n \\ &= \sum_{j=0}^{\infty} j P_{s+j} \\ &= \sum_{j=0}^{\infty} j \frac{(\lambda/\mu)^{s+j}}{s! s^j} P_0 \\ &= \frac{(\lambda/\mu)^s}{s!} P_0 \sum_{j=0}^{\infty} j \left(\frac{\lambda}{s\mu}\right)^j \\ &= \frac{(\lambda/\mu)^s}{s!} P_0 \sum_{j=0}^{\infty} j \rho^j \\ &= \frac{(\lambda/\mu)^s}{s!} P_0 \rho \sum_{j=0}^{\infty} j \rho^{j-1} \\ &= \frac{(\lambda/\mu)^s}{s!} P_0 \rho \frac{d}{d\rho} \sum_{j=0}^{\infty} \rho^j \\ &= \frac{(\lambda/\mu)^s}{s!} P_0 \rho \frac{d}{d\rho} \frac{1}{1-\rho} \\ &= \frac{(\lambda/\mu)^s}{s!} P_0 \frac{\rho}{(1-\rho)^2} \end{aligned}$$