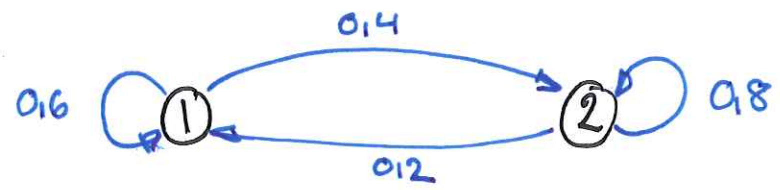
Ex: Consider a production unit that can be in two states.

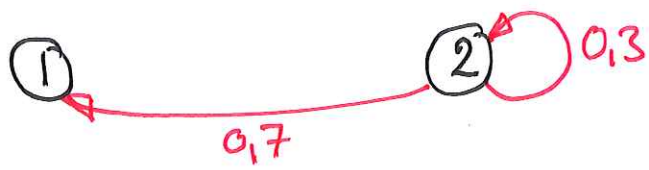
State 1	State 2
The unit is working WELL generates income of 400 \$ per week	The unit is working POORLY generates income of 250 \$ per week.

Without maintenance the transition probabilities are



If we do maintenance in state 1 the cost is 50 \$ and change the transition probabilities to

If we do maintenance in state 2 the cost is 200 \$ and change the transition probabilities to



Determine optimal maintenance policy.

"When should we do maintenance?"

<u>Aim:</u> Determine decision policy R that determine an action that only depends on the current state

Let $d(R) = \begin{bmatrix} d_1(R) \\ d_2(R) \end{bmatrix}$

where $d_i(R) = \begin{cases} 1 & \text{if maintenance is done in state } i \\ 2 & \text{if maintenance is not done in state } i \end{cases}$

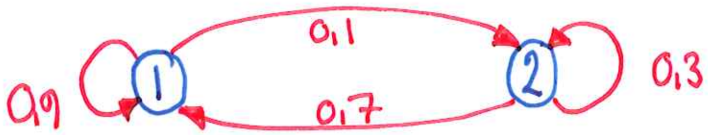
There are 4 different policies in our case.

$d(R_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad d(R_2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad d(R_3) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad d(R_4) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Each policy corresponds to a different Markov Chain.

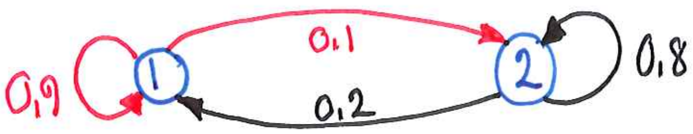
<u>Transition matrices $P(k)$</u>

R_1:



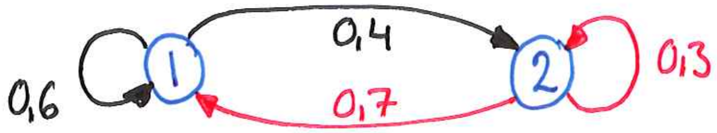
$$P(1) = \begin{bmatrix} 0{,}9 & 0{,}1 \\ 0{,}7 & 0{,}3 \end{bmatrix}$$

R_2:



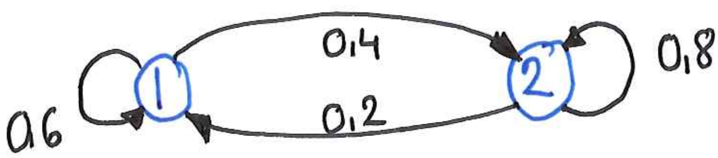
$$P(2) = \begin{bmatrix} 0{,}9 & 0{,}1 \\ 0{,}2 & 0{,}8 \end{bmatrix}$$

R_3:



$$P(3) = \begin{bmatrix} 0{,}6 & 0{,}4 \\ 0{,}7 & 0{,}3 \end{bmatrix}$$

R_4:



$$P(4) = \begin{bmatrix} 0{,}6 & 0{,}4 \\ 0{,}2 & 0{,}8 \end{bmatrix}$$

Objective: Maximize expected profit

income - depends on state
cost - depends on maintenance decision

Let C_{ik} = Expected value of immediate cost incurred by making decision $d_i = k$ in state i.

$C_{11} = \{$ in state 1, maint. $\} = -400 + 50 = -350$

$C_{12} = \{$ in state 1, no maint. $\} = -400 + 0 = -400$

$C_{21} = \{$ in state 2, maint. $\} = -250 + 200 = -50$

$C_{22} = \{$ in state 2, no maint. $\} = \underbrace{-250}_{\text{income}} + \underbrace{0}_{\text{cost}} = -250$

We need: Stationary distributions corresponding to each policy

Steady state equations $\pi(k) = P(k)\pi(k)$, $\sum_{i=1}^{2} \pi_i(k) = 1$

gives $\pi(1) = [7/8 \quad 1/8]$ $\quad \pi(2) = [2/3 \quad 1/3]$

$\pi(3) = [7/11 \quad 4/11]$ $\quad \pi(4) = [1/3 \quad 2/3]$

Stationary expected cost $F(R_k) = \sum_{i=1}^{2} C_{i,d_i(R_k)} \cdot \pi_i(k) \quad k = 1 \ldots 4$

$F(R_1) = C_{11}\pi_1(1) + C_{21}\pi_2(1) = (-350)\frac{7}{8} + (-50)\frac{1}{8} = -312{,}5$

$F(R_2) = C_{11}\pi_1(2) + C_{22}\pi_2(2) = (-350)\frac{2}{3} + (-250)\frac{1}{3} = \underline{-316{,}7}$

$F(R_3) = C_{12}\pi_1(3) + C_{21}\pi_2(3) = (-400)\frac{7}{11} + (-50)\frac{4}{11} = -272{,}7$

$F(R_4) = C_{12}\pi_1(4) + C_{22}\pi_2(4) = (-400)\frac{1}{3} + (-250)\frac{2}{3} = -300$

$\therefore$ Policy R_2 is the best!