<u>Assume:</u> $p_{ij} = p_i^1 p_j^2 = \left[\left(1-\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^i\right]\left[\left(1-\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\mu_2}\right)^j\right]$

<u>Show:</u> $p_{i-1,j}\,\lambda + p_{i+1,j-1}\,\mu_1 + p_{i,j+1}\,\mu_2 = p_{ij}\,(\lambda+\mu_1+\mu_2)$

$$\text{LHS} = \left(1-\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^{i-1}\left(1-\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\mu_2}\right)^{j}\lambda + \left(1-\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^{i+1}\left(1-\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\mu_2}\right)^{j-1}\mu_1 + \left(1-\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^{i}\left(1-\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\mu_2}\right)^{j+1}\mu_2$$

$$= \left(1-\frac{\lambda}{\mu_1}\right)\left(1-\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\mu_1}\right)^{i-1}\left(\frac{\lambda}{\mu_2}\right)^{j-1}\underbrace{\left[\left(\frac{\lambda}{\mu_2}\right)\lambda + \left(\frac{\lambda}{\mu_1}\right)^2\mu_1 + \left(\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_2}\right)^2\mu_2\right]}_{=\frac{\lambda}{\mu_1}\cdot\frac{\lambda}{\mu_2}\left[\mu_1+\mu_2+\lambda\right]}$$

$$= \left(1-\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^{i}\cdot\left(1-\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\mu_2}\right)^{j}(\mu_1+\mu_2+\lambda) = p_{ij}\,(\lambda+\mu_1+\mu_2) = \text{RHS.}$$