Errata for the Exercises in SF2863

Corrected versions are given below:

1.8

Clarification: In state $S2$ we have first received a 0 and then a 1.
(so the next state will first have received a 1 and then a 0 or a 1)

2.1

We assume that the visit starts at the carousel.

2.2c answer

Only 50% of all customers passing the kiosk buys the aspirin, so the expected income per hour should be $10 \cdot \lambda_K \cdot 0.5 = 40$ Kr.

2.5

The expected value for the time until a computer crashes is 120 *hours*.

In **(a)**, The requirements that are set up by the department is that the *mean time down* for a crashed computer must not be larger than 13 hours.

In **(b)**, the answer, there are some rounding errors. $L \approx 0.3677$, $\bar{\lambda} \approx 0.0303$, $W \approx 12.15$.

In **(c)**, "the *mean time* before it receives service must not exceed four hours."

In **(c)**, the answer, there are some rounding errors. $W_q = W - \frac{1}{\mu} \approx 0.15$ and $\frac{0.15}{0.0042} \approx 34.61$.

4.1 answer

We have $V_3(x_3) = u_3^2 = (2x_3)^2 = 4x_3^2$, since $u_3 = 2x_3$ is needed for $x_4 = 0$.
Recursion equation: $V_k(x_k) = \min_{u_k}\{u_k^2 + V_{k+1}(2x_k - u_k)\}$ for $k = 2, 1, 0.$.
From above $V_3(x_3) = c_3x_3^2$ with $c_3 = 4$.

In fact, it is possible to find a general expression for the solutions to the recursion equation.

Assume that $V_{k+1}(x_{k+1}) = c_{k+1}x_{k+1}^2$ for some fixed $k \geq 0$. From the recursion equation we then have that $V_k(x_k) = \min_{u_k}\{u_k^2 + c_{k+1}(2x_k - u_k)^2\}$, and minimization w.r.t. u_k gives $\hat{u}_k = 2c_{k+1}/(1 + c_{k+1})x_k$. Knowing the expression for the optimal control, we get that

$$V_k(x_k) = \left(\frac{2c_{k+1}}{1 + c_{k+1}}x_k\right)^2 + c_{k+1}\left(2x_k - \frac{2c_{k+1}}{1 + c_{k+1}}x_k\right)^2 = 4\frac{c_{k+1}}{1 + c_{k+1}}x_k^2 = c_kx_k^2$$

where $c_k = 4c_{k+1}/(1 + c_{k+1})$.

The assumed structure does actually hold.
The constants c_k are determined from the recursion:

$$c_3 = 4, \quad c_2 = \frac{4c_3}{1+c_3} = 16/5, \quad c_1 = 64/21, \quad c_0 = 256/85.$$

This gives the optimal controls: ($\hat{u}_k = c_k/2x_k$)

$$\hat{u}_3 = 2x_3, \quad \hat{u}_3 = 8x_2/5, \quad \hat{u}_1 = (32/21)x_1, \quad \hat{u}_0 = (128/85)x_0,$$

The total cost is $V_0(85) = (256/85) * 85^2 = 21760$.
Check: $16^2 + 32^2 + 64^2 + 128^2 = 21760$.

4.2 (b) answer
It should be $v_3(1) = c_2 + q$
In expression for $v_4(1)$, $c_1 + q + v_3(1) = c_1 + c_2 + 2q$.
In expression for $v_4(0)$, $c_2 + q + v_3(1) = 2c_2 + 2q$.

4.4, 4.5 The answers to these exercises are missing, but Mikael has solved these problems in the exercise classes.

5.2
Part of the formulation got lost in translation.
"If Luckybet does not take her husband out for dinner, Fluke will be in a good mood the following Saturday with probability 1/8 and in a bad mood with probability 7/8."