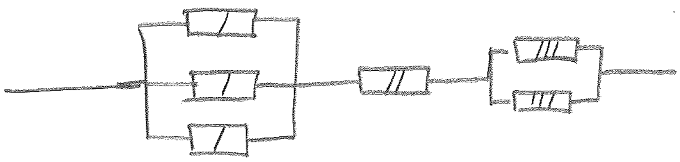
Reliability system design

Electromechanical system with three components in series



Reliability for components:

Reliability of system = product of reliability for each component.

$p(x) = p_1(x_1)\,p_2(x_2)\,p_3(x_3)$

Probability of functioning $p_i(x_i)$

Number of parallell units x_i	component $i=1$	component $i=2$	component $i=3$
1	0,6	0,7	0,8
2	0,7	0,8	0,9
3	0,9	0,9	0,95

Cost for installing components:

Budget limit 10 k$.

Cost of component $c_i(x_i)$

Number of parallell units x_i	component $i=1$	component $i=2$	component $i=3$
1	3	2	1
2	5	4	4
3	6	5	5

Choose x_1, x_2, x_3 so that the reliability of the system is maximized.

$$\left[\begin{array}{l} \max \;\; p_1(x_1)\,p_2(x_2)\,p_3(x_3) \\ \text{s.t.} \begin{cases} c_1(x_1)+c_2(x_2)+c_3(x_3) \le 10 \\ x_1, x_2, x_3 \in \{1,2,3\}. \end{cases} \end{array}\right]$$

at stage 2

$$\max \;\; p_2(x_2)\,p_3(x_3)$$
$$c_2(x_2)+c_3(x_3) \le S$$
$$x_2, x_3 \in [1,2,3]$$

Dyn.P. structure: At stage i, components $x_i, \dots, x_3$ are considered.

The remaining budget is s_i k$ (the state).
and the number of components x_i is next to be determined.

$\Rightarrow \; s_{i+1} = s_i - c_i(x_i)$ is the remaining budget at next stage

Let $f_i^*(s_i)$ = maximal probability of a functioning subsystem with components $i, \dots, 3$ with budget constraint for these components is s_i.

$$f_i^*(s_i) = \max_{x_i \in \{1,2,3\}} \left\{ p_i(x_i) \cdot f_{i+1}(s_i - c_i(x_i)) \right\}.$$

$$f_3^*(s_3) = \max_{x_3 \in \{1,2,3\}} \left\{ p_3(x) \; : \; c_3(x_3) \le s_3 \right\}.$$

Start at stage 3:

Budget surplus. → s_4

s_3	$p_3(x_3)$: $x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$f_3^*(s_3)$	x_3^*	s_4
1	0,8	—	—	0,8	1	0
2	0,8	—	—	0,8	1	1
3	0,8	—	—	0,8	1	2
4	0,8	0,9	—	0,9	2	0
5	0,8	0,9	0,95	0,95	3	0
6	0,8	0,9	0,95	0,95	3	1
7	0,8	0,9	0,95	0,95	3	2
8	0,8	0,9	0,95	0,95	3	3
9	0,8	0,9	0,95	0,95	3	4
10	0,8	0,9	0,95	0,95	3	5

$10 - c_2(1) - c_1(1)$

$= 10 - 2 - 3 = 5$

We have to have at least $x_1 = 1, x_2 = 1$ so at most we have 5 k$ left at stage 3

Stage 2:

s_2	$p_2(x_2) \cdot f_3(s_2 - c_2(x_3))$: $x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$f_2^*(s_2)$	x_2^*	s_3
1	—	—	—			
2	—	—	—		1	0
3	$0,7 \cdot 0,8 = 0,56$	—	—	0,56	1	1
4	$0,7 \cdot 0,8 = 0,56$	—	—	0,56	1	2
5	$0,7 \cdot 0,8 = 0,56$	$0,8 \cdot 0,8 = 0,64$	—	0,64	2	1
6	$0,7 \cdot 0,9 = 0,63$	$0,8 \cdot 0,8 = 0,64$	—	0,64	2	2
7	$0,7 \cdot 0,95 = 0,665$	$0,8 \cdot 0,8 = 0,64$	$0,9 \cdot 0,8 = 0,72$	0,72	3	2
8						
9						
10						

$s_3 \geq c_3(1) = 1$

We have to have $s_2 \geq 3$ to buy at least $x_2 = 1$ and $x_3 = 1$.

$10 - c_1(1) = 7$

We have at most 7 k$ left at stage 2.

Stage 1:

s_1	$p_1(x_1) f_2(s_1 - c_1(x_1))$: $x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$f_1^*(s_1)$	x_1^*	s_2
10	$0,6 \cdot 0,72 = 0,432$	$0,7 \cdot 0,64 = 0,448$	$0,9 \cdot 0,56 = 0,504$	0,504	3	4
	$s_1 - c_1(1) = 7$ $10 - 3$	$s_1 - c_1(2) = 5$ $10 - 5$	$s_1 - c_1(3) = 4$ $10 - 6$			

Optimal solution ⇒ reliability = 0,504

$s_1 = 10$ $x_1^* = 3$ $s_2 = 4$

$s_2 = 4$ $x_2^* = 1$ $s_3 = 2$

$s_3 = 2$ $x_3^* = 1$ $s_4 = 1$

↑ We have 1 k$ extra at the end!

	Dyn P	MA.
pro:	Do not need convexity	Efficient calculations