KTH Mathematics

Exam in SF2863 Systems Engineering
Thursday March 13, 2014, 8.00–13.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.
No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, and always define the variables and notation you use. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. This Olympic year Frasse has been working a lot. Even if it is more important to participate than to win, it is essential to optimize. Now, Frasse is considering to, once again, despite previous experiences, employ his brother-in-law Heathcliff.

Frasse is running an office where a receptionist is receiving calls, which is modelled as a $M|M|1$ queue with arrival intensity 1 customer per hour and service intensity 10 customers per hour. After the administrative handling has been taken care of the receptionist transfer the callers to Frasse. The consulting service of Frasse is also modelled as a $M|M|1$ queue with service intensity 1.2 customers per hour. After Frasse has finished his service the customer leaves the system.

Determine the average time it takes for a customer calling Frasse's office until he leaves the system.

If Heathcliff is employed he would help out with the consulting service in the following way. When the receptionist transfer a call it will with probability p go to Frasse and with probability $1-p$ to Heathcliff. The consulting service of Heathcliff is also modelled as a $M|M|1$ queue with service intensity 2 customers per hour. However, with probability $1/2$ the customer will not be helped by Heathcliff and is then routed to the inbound queue for Frasse and will be served by him as if they have not visited Heathcliff. With probability $1/2$ they are actually helped and leave the system.

Determine the average time, as a function of p, it takes for a customer calling Frasse's office until he leaves the system when Heathcliff is employed.

Determine the value of p that minimizes the time for the customer to leave the system, *i.e.*, the probability the receptionist should use when connecting the customers.

What is the probability that the whole system is empty? Determine this both for the case with Heatcliff employed and without.

If the average caller to the receptionist has a salary of 100 dollars per hour and are employed by the same company as Frasse. From a company viewpoint, should Frasse employ Heathcliff if he is willing to do the job for 50 dollars per hour? ...... (12p)

2. Frasse has been asked by the swedish ski federation to help with the strategic planning for the olympic 50 km cross-country ski race. In the 50 km race the skiers have the option after each 10 km of the race to change their skis.

His assignment is to determine when and if the swedish skiers should change their skis in order to achieve the best finishing time. According to Frasse's (very simplified) model of the swedish skiers it takes them 25 minutes to complete a 10 km lap on fresh skis. A change of skis, to a fresh pair, takes the skier an extra 30 seconds. If the skier do not change the skis, it will take him an extra 20 seconds to complete the next lap for each 10 km the skier has already used the current skis before starting the current lap.

Formulate the optimization problem that Frasse is supposed to solve. Define carefully the variables you introduce, and the objective function that he wants to minimize.

Determine a dynamic programming recursion for the optimal remaining objective function.

Solve the problem for the 50 km race where we assume the skier starts with fresh skis. .... (8p)

3. Frasse has been hired to organize the training of the swedish biathlon relay team to the next olympics. The team has already been set and there are four athletes that will take part of the team. They will all have individual training programs and the training resources should be distributed in such a way that the total performance of the team is maximized, *i.e.*, the total finish time is minimized. To make it simple, Frasse has defined units of resources that can be assigned to the athletes, and each athlete can be assigned 0,1,2,3 or 4 resources, and those resources will be fixed during the whole training period.

According to Frasses estimations, the individual finish times of the four athlete are given by the functions

$$d_1^2 - 10d_1 + 30, \quad d_2^2 - 11d_2 + 36, \quad d_3^2 - 12d_3 + 44, \quad d_4^2 - 13d_4 + 50,$$

where d_i are the number of training resources that athlete i is assigned.

Use the marginal allocation algorithm (and show that the conditions for using it are satisfied) to determine all efficient solutions for maximizing the performance of the team when the total number of available training resources ranges in $\{0, \cdots, 6\}$.

When you present the efficient solutions we want both the minimizing estimated finish times and the distributions of the training resources used to achieve that performance. .... (6p)

4. (a) As a side business Frasse is selling tickets to the games. He can buy the tickets for 60 dollars each through a contact in the organization. His plan is that Heathcliff is going to sell the tickets for 100 dollars each. According to the research Frasse has performed the number of tickets Heathcliff should be able to sell before the game starts is determined by a stochastic variable with a uniform distribution on the integers $\{40, \cdots 100\}$. Any tickets not sold at the start of the game can be sold for 10 dollars each to the economy minded sportfantasts waiting for the prices to be dumped.

How many tickets should Frasse buy in order to maximize his expected profit from this side business.

.... (6p)

(b) As another side business Frasse is selling exclusive cars in the exhibition center. Each time he orders a delivery of cars one truck, (or several trucks) with capacity of up to four cars is sent with the delivery and the delivery cost is 5.000 dollars per truck. All the delivered cars that are not sold yet, are kept in the exhibition room and Frasse has to pay a show cost of 1.000 dollars per car and day to the exhibition center. The delivery arrives at the beginning of the day and if it is sold the same day there is no show cost.

In fact, all the cars that Frasse sells are pre-ordered so he knows that he will sell d_k cars day k.

Assume that each time a new delivery with cars arrive there are no cars remaining in the exhibition room.

Determine a dynamic programming recursion for the optimal cost, under the assumption above, for a period of N consecutive days, where you should carefully define all variables.

Determine the optimal order plan when $N = 4$ and $d_1 = 3$, $d_2 = 3$, $d_3 = 1$, $d_4 = 2$.

Show that it is not always optimal to empty the inventory before ordering new cars for other order plans than the one given here.

.... (10p)

5. Heathcliff, the famous brother-in-law of Frasse, and life member of the world curling association, has mysteriously managed to get Frasse the job to sell the broadcasting rights for the olympic games for all future. Every four years there is an occasion where the rights are being sold.

The selling model that Frasse is going to use is to start the auction with a low or a high price. The outcome will be random but depends on the outcome of the last olympic game.

If the outcome of the last olympics was better than expected then

- If the starting price is low then with probability 0.8 the outcome will be better than expected and generate a net income of 2 billion dollars, and with probability 0.2 the outcome will be worse than expected and generate a net income of 1 billion dollars.

- If the starting price is high then with probability 0.5 the outcome will be better than expected and generate a net income of 3 billion dollars, and with probability 0.5 the outcome will be worse than expected and generate a net income of 1/2 billion dollars.

If the outcome last olympics was worse than expected then

- If the starting price is low then with probability 0.6 the outcome will be better than expected and generate a net income of 1 billion dollars, and with probability 0.4 the outcome will be worse than expected and generate a net income of 1/2 billion dollars.
- If the starting price is high then with probability 0.4 the outcome will be better than expected and generate a net income of 2 billion dollars, and with probability 0.6 the outcome will be worse than expected and generate a net income of 1 billion dollars.

Start with the policy to always use a high starting price. Use the policy iteration algorithm (without discounting) to *improve on* the starting policy, if possible.
Is the starting policy optimal? If not, determine a policy that gives a higher expected net income per olympic game at steady state. What is the expected income for each olympic game at steady state for the best policy you have obtained? .... (8p)

Good luck!