KTH Mathematics

Exam in SF2863 Systems Engineering
Monday January 11, 2016, 14.00–19.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.

No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, and always define the variables and notation you use. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. Frasse is battling the global warming and pollution in the world by minimizing the environmental effects of a next generation car model. The car manufacturer has to decide which type of engine, which type of catalytic converter, and which materials to use in the new car model. The environmental effects of the three categories of actions are measured by the function $E(\mathbf{n}) = E_e(n_e) + E_c(n_c) + E_m(n_m)$, $\mathbf{n} = (n_e, n_c, n_m)$, where a low value indicates a low environmental effect and n_e, n_c, n_m are the levels of actions, described in the following table:

n	$E_e(n)$	$E_c(n)$	$E_m(n)$
0	12	15	18
1	7	9	15
2	5	6	13
3	4	4	12

If there were no budget constraint the best solution would be to have the highest level of each action. Frasse has been asked to find all efficient solutions such that the budget of 60.000 SEK is not breached. The costs are given in the table below, in kSEK, thousands of SEK.

n	$c_E(n)$	$c_c(n)$	$c_m(n)$
0	14	10	20
1	16	12	24
2	24	20	30
3	36	34	44

Use the Marginal allocation algorithm (and show that the conditions for using it are satisfied) to determine all efficient solutions.

When you present the efficient solutions we want both the level of each action, and the provided environmental effect and the costs. .... (8p)

2. Frasse has invented a revolutionary gadget that will solve all the energy problems in the world. He will now file a patent and is trying to determine the average time it will take until he can get rich, and how much it will cost him.

First he has to write a patent paper together with a patent lawyer. This process is modelled as a $M|M|1$ queue, where the Poisson process describing the arrival rate of new clients to the lawyer has intensity 10 clients per month, and the service rate is 16 clients per month.

Next the paper is sent to a patent office, specially dedicated only to the customers of this patent lawyer, which is modelled as an $M|M|2$ queue with service rate 10 clients per month and server.

With probability 0.2 the patent is rejected, with probability 0.3 the patent is returned to the patent lawyer, and with probability 0.5 the patent is accepted.

How much time will it take on average from first contact to the patent is either rejected or accepted?

What is the probability that it will be finally rejected or accepted?

What is the average total cost if the patent lawyer and the patent office both takes 500 dollars per day, assuming there are 24 working days each month.
Assume that the cost is linear, so you only pay for the fraction of a day that you get service. So the cost is NOT *for each started hour or day, and is* NOT *charged for time queueing.* .... (10p)

3. Frasse is asked to optimize the profit of a sawmill. The mill receives lumber of different length and then cuts it into specified lengths $\ell_1, \cdots, \ell_N$ which can be selled for fixed prices $p_1, \cdots, p_N$ each.

Assume that the mill receives lumber of length L, then Frasse should solve the following optimization problem:

$$\left[\begin{array}{ll} \max\limits_{x_1,\cdots,x_N} & \sum_{i=1}^{N} x_i p_i \\ \text{s.t} & \sum_{i=1}^{N} x_i \ell_i \leq L \\ & x_i \in \{0, 1, 2, \cdots\} \text{ for } i = 1, \cdots, N. \end{array}\right]$$

where x_i denote the number of pieces of length ℓ_i that are made of each log of length L.

Use a dynamic programming approach to solve the problem.
Determine a recursive formula for the optimal values of problems of this kind.

Define carefully the variables and functions that you introduce.

Solve the problem for the case $L = 4$,

$$p_1 = 2, p_2 = 5, p_3 = 7, p_4 = 9,$$

and

$$\ell_1 = 1, \ell_2 = 2, \ell_3 = 3, \ell_4 = 4,$$

i.e., determine the optimal value and optimal cutting strategy. ....(10p)

4. (a) Frasse is driving an old Mercedes, which he estimates will last 10 more years. Assume that each year has 50 active weeks. His current fuel costs are 500 dollars per year, or 10 dollars per week. He also has to pay repair costs and rental car costs for when the car breaks down.

The probability that the car breaks down is 5%, each week. If it breaks down there is a repair cost of 200 dollars (one-time cost) and he has to pay 50 dollars per week for a rental car. (The fuel consumption of the rental car is assumed to be included in the 50 dollars. The week that the Mercedes breaks down the rental car cost has to be paid but the fuel cost for the Mercedes does not have to be paid.)

The probability that the car is fixed is 80%, each week it is in the repair shop. Consider this model as a Markov process where the state $x_i = 1$ if the car is working in the i:th week $i = 1, 2, \cdots, 500$, and zero if it is not working.

Determine the total cost for the 10 years by assuming that the process is in steady state from the start. ....(5p)

(b) Now assume that Frasse can use an alternative fuel that has half the price but increase the risk of breakdown to 10%, and he can let Heatcliff try to repair the car for 50 dollars but then the success rate each week is only 40 %. The alternative fuel is only used in the Mercedes. Then there are two alternative decisision that can be made in each state.

Use the policy improvement algorithm, starting with the policy corresponding to the problem in (a), and compute an updated policy and determine the expected cost it corresponds to. ....(7p)

5. (a) Frasse has a new startup company "Pop protein" specialized on selling frozen grasshoppers. His trusted supplier is of course his brother in law Heathcliff, who charges 100 dollars for each delivery and 3 dollars per kilo. Frasse has already established contracts with five restaurants who generate a total demand rate of 100 kilos per day. The storage cost is 0.05 dollars per kilo and per day. Shortage is not allowed. Determine how often Frasse should order and how much he should order each time if he wants to minimize the average cost per time unit. ....(4p)

(b) Assume now that Frasse refines his storage cost model so that the storage cost is 5 dollars per day for each freezer he is using, where each freezer has a capacity of 100 kilos. In fact, he decides that each time he orders he will order a multiple of 100 kilos.

Determine the average cost per time unit for this new problem. Show that this function is integer-convex and determine the optimal order quantity. ....(6p)

Good luck!