KTH Mathematics

Exam in SF2863 Systems Engineering
Wednesday March 16, 2016, 8.00–13.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.

No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, and always define the variables and notation you use. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the questions does not correspond to their difficulty level.

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1. Heathcliff, Frasse's beloved brother-in-law, was born in a land of opportunities and this year he has decided to run for president. His budget is somewhat limited, so therefore he has asked Frasse to help him to choose which lobby organizations to contract.

The expected positive effect of lobby organization i, depending on the level of contract n_i, are measured by the function $L(\mathbf{n}) = L_1(n_1) + L_2(n_2) + L_3(n_3)$, $\mathbf{n} = (n_1, n_2, n_3)$, where a low value indicates a low effect and n_1, n_2, n_3 are the levels of actions, described in the following table:

n	$L_1(n)$	$L_2(n)$	$L_3(n)$
0	0	0	0
1	10	15	15
2	15	17	18
3	18	19	20

Frasse has been asked to find all efficient solutions such that the budget of 50.000 \$ is not exceeded. The costs are given in the table below, in k\$, thousands of \$.

n	$c_1(n)$	$c_2(n)$	$c_3(n)$
0	0	0	0
1	6	7	8
2	21	17	17
3	38	28	28

Use the Marginal allocation algorithm, and show that the conditions for using it are satisfied, to determine all efficient solutions.

When you present the efficient solutions we want both the level of each action, the expected positive effect and the costs.

Is the order of the efficient solutions unique?
How do different budget constraints impact the choice of efficient solutions? .(10p)

2. Frasse is also responsible for the communication system and staffing at the campaign headquarters. All the incoming calls to the headquarters arrive to an automatic router of the type press 1 to become a member, press 2 to discuss politics, or press 3 to donate money. The incoming call intensity is 100 calls per hour and Frasse assumes that it is well approximated with a Poisson process. He also assumes that 30% of all callers press 1, 50% press 2 and 20% press 3, and the time required to perform this action is assumed to be exactly 30 seconds. For each choice the caller is connected to a queueing system, that Frasse choose to approximate with a $M|M|1$ queue. An $M|M|s$ queue would be more appropriate, but since Frasse thinks the formulas are a bit complicated he use an $M|M|1$ queue with service intensity $s_i\mu_i$, where s_i are are the number of servers at service station i and $1/\mu_i$ is the mean value of the service time for each server.

Callers who have finished service at station 1 is with 20% probability routed back to station 2 "discuss politics", and callers who have finished service at station 2 is with 50% probability routed back to station 3 "donate money". Everyone else exits the system.

Determine the least number of servers s_1, s_2, s_3 required for the system to reach steady state, if $\mu_1 = 30$, $\mu_2 = 10$ and $\mu_3 = 20$ clients per hour.

What is the average time to pass through the system for that choice of number of servers? (Using the $M|M|1$ approximation described above)

What are the largest problems with approximating an $M|M|s$ queue with an $M|M|1$, as used here. .... (10p)

3. Frasse is also responsible for the advertisement. He should order the sizes of campaign posters to be published at various locations. Let $x_1, \cdots, x_N$ be the sizes of the posters placed at locations $1, \cdots, N$. Frasse believes that the impact of placing a poster of size x_i at location i is given by $I_i x_i^2$. There is also a cost for posting a poster of size x_i at location i which is given by $c_i x_i$.

The optimization problem Frasse wants to solve is now given by

$$\left[\begin{array}{ll} \max\limits_{x_1,\cdots,x_N} & \sum_{i=1}^{N} I_i x_i^2 \\ \text{s.t} & \sum_{i=1}^{N} c_i x_i \leq C \\ & x_i \geq 0 \text{ for } i = 1, \cdots, N. \end{array}\right]$$

Use a dynamic programming approach to solve the problem.
Determine a recursive formula for the optimal values of problems of this kind.

Define carefully the variables and functions that you introduce.

Solve the problem for the case $N = 3$, $C = 10$

$$I_1 = 1, I_2 = 3, I_3 = 4,$$

and

$$c_1 = 1, c_2 = 2, c_3 = 1,$$

i.e., determine the optimal value for the impact and the optimal poster sizes. (10p)

Alternatively, it was allowed to consider the optimization problem:

$$\left[\begin{array}{ll} \min\limits_{x_1,\cdots,x_N} & \sum_{i=1}^N I_i x_i^2 \\ \text{s.t} & \sum_{i=1}^N c_i x_i = C \\ & x_i \geq 0 \text{ for } i = 1, \cdots, N. \end{array}\right]$$

4. As the technical political advisor of Heathcliff, Frasse is coaching him to improve his number of votes. Frasse is modeling the popularity of Heathcliff with a Markov decision process where the state s_k is 2 if Heathcliff is hot and been a lot in the media since election k and s_k is 1 if he is not.

In the model of Frasse, if Heathcliff is hot he gets 50 votes and if not he only gets 10 votes. The aim of the coaching by Frasse is to maximize the number of total votes that Heathcliff gets.

Under normal political conditions the probability that Heathcliff remains hot from one stage to the next is 60%, (the probability that he goes from hot to not is then 40%), and that he remains not hot from one stage to the next is 75%.

Frasse can change these transition probabilities by coaching Heathcliff to make a controversial political statement. The probability that Heathcliff remains hot from one stage to the next is then 50%, and that he remains not hot from one stage to the next is 40%. However, each time he makes such a statement the model reduces the number of total votes by 40 if Heathcliff is not hot, since voters might find him desperate and loose longterm confidence, and increases the number of votes by 20 if Heathcliff is hot because then the voters might find him determined.
Note that the these votes are artificially defined in the model to take in to account the voters interpretations of the candidates credibility.

Use the policy improvement algorithm, starting with the policy to not do any controversial statements. Compute an updated policy and determine the expected number of votes it corresponds to. .... (10p)

5. Another part of the marketing is the sales of “Heathcliff for President” T-shirts that Frasse is selling. He has done some consumer tests and is now quite sure that the demand D of T-shirts is well described by a triangular distribution, *i.e.*,

$$\Pr(D = k) = \begin{cases} k/L & k = 0, 1, \cdots, N \\ (2N - k)/L & k = N + 1, \cdots, 2N \end{cases}$$

where $N = 500.000$ and $L = N^2$.

The supplier of Frasse charges 3 dollars per T-shirt, Frasse will sell them for 10 dollars per T-shirt and the salvage value per T-shirt after the election is 1 dollar.

How many T-shirts should Frasse order if he wants to maximize the expected profit. .... (10p)

Good luck!