

## Exam in SF2863 Systems Engineering Thursday April 9, 2015, 8.00–13.00

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*Allowed tools:* Pen/pencil, ruler and eraser. A formula sheet is handed out. No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, and always define the variables and notation you use. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!

25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.

The order of the exercises does not correspond to their difficulty level.

1. Frasse is going to a football game with his niece Celia in the Parc des Princes arena. He is trying to figure out how much time it will take on average from arriving at the arena until they are in their seats.

The way Frasse decides to model the arena is the following. Based on 48000 seats in the sold out arena, and an estimate of 48 different sectors with a separate entrance each he assumes that the sector they are going through is fed by a Poisson process with arrival intensity 1000 persons per hour. He model the entrance service as a one server facility with service time that is exponentially distributed with a mean service time of 3 seconds.

Then there is a five minute walk to get into the arena and the vending area. Frasse predicts that there is a probability of 0.1 that Celia wants to buy a souvenir, a probability of 0.2 that she wants to buy a snack and a probability of 0.7 that they will go directly to their seats.

The service of the souvenir shop is from a single server with exponentially distributed service time with expected value 20 seconds. When Celia exits the souvenir shop she wants to have a snack with probability 0.2 and go to the seats with probability 0.8

The service of the snacks shop is from two servers with exponentially distributed service time with expected value 30 seconds each. When Celia exits the snacks shop she wants to have a souvenir with probability 0.3 and go to the seats with probability 0.7.

In fact, Frasse assumes that all the football fans have the same behaviour as Celia and act independently. (Except for himself, who just follows Celia around.)

From the vending area there is a two minutes walk to the seats.

What is the average total time it takes to pass through the system at steady state? (From arriving at the end of the queue to beeing seated)

What is the average total number of people in the system at steady state? (Including the persons walking between the different stations)

2. Chelsea and PSG are about to play two important games to decide which team qualifies for the next round of the cup. In the battle of minds, Chelseas coach Mourinho has consulted Frasse to determine the best strategy to deal with PSGs superstar Zlatan Ibrahimovic.

In a normal game, Chelsea would win the first game with probability 0.3, draw with probability 0.3 and loose with probability 0.4. Then, if Chelsea wins the first game they win the second with probability 0.5, draw with probability 0.3 and loose with probability 0.2. If Chelsea draws the first game they win the second with probability 0.4, draw with probability 0.3 and loose with probability 0.3. If Chelsea looses the first game they win the second with probability 0.4 and loose with probability 0.3.

For each game, Chelsea can try to provoke Zlatan, in which case the probability of winning and loosing the game increase by 0.1, hence the probability of a draw is reduced by 0.2.

A win gives 3 points, a draw gives 1 point, a loss 0 points, and the most total points qualifies for the next round. If they have equally many points after the two games, then Frasse models the probability to advance as 0.5 for each team.

Use a dynamic programming approach to solve the problem to find the strategy for the two games that maximizes the probability for Chelsea to advance to the next round.

Define carefully the variables you introduce, and the objective function considered.

3. The silly-season is on and the word is going around in the corridors of the big arenas about Frasses great analytical skills. He is approached by a manager who wants help to buy the right players for the next season. The team needs a new defender, one midfielder and one forward. There are a few candidates and Frasse has ranked them as 1-star, 2-star or 3-star players and has formed a utility function describing the positive effect that the player would have for the team, and it is defined as  $u(n) = u_D(n_D) + u_M(n_M) + u_F(n_F)$  where the utilities are given by

n	$u_D(n_D)$	$u_M(n_M)$	$u_F(n_F)$
1	5	3	7
2	8	5	12
3	10	6	15

depending on the number of stars the player bought has, defined by  $n = (n_D, n_M, n_F)$ . The corresponding costs for buying the 1-star, 2-star or 3-star players are given by

n	$c_D(n_D)$	$c_M(n_M)$	$c_F(n_F)$
1	0.3	0.3	0.5
2	0.6	0.6	0.8
3	1.0	0.9	1.2

Use the Marginal allocation algorithm (and show that the conditions for using it are satisfied) to determine all efficient solutions for maximizing the utility given that the budget is in the range [1.1, 2.3] million SEK. Start with 1-star players at all three positions, then for increasing budget, corresponding to efficient solutions, determine how many stars the player at each position should have.

(There should never be more than one player at each position)

4. One of the most well hidden secrets in the world of football is that Mino Raiola, the player agent of Zlatan Ibrahimovic, has been consulting Frasse for strategic advice. Frasse has been determining the optimal strategy for when the player should change team.

The model that Frasse base his analysis on is the following, which may or may not be different from what you think is going on in the real football world. All his players have 3 year contracts. Assume that the player starts the year with a new contract. At the end of the first year the player may either continue his contract or renegotiate a new 3 year contract. At the end of the second year (on the current contract) the player may either continue his contract. At the end of the third year (on the current contract) the player may either contract) the player may either contract.

Consider this model as a Markov Decision process where the state  $x_t = i$  if the player is in the *i*:th year of the contract, i = 1, 2, 3.

Determine all the possible decision policies.

Classify the states of the Markov process for the different decision policies.

The estimates of Raiola for the income for the first year on a contract for Zlatan Ibrahimovic is 4 million Euro, for the second year 3, and the last 3. If Zlatan decides to change contract after the first or second year he will get another 2 million Euro in income (sign on bonus minus release fee and moving expenses). If Zlatan stays for the full time of the contract he will be able to avoid release fees, lex Bosman, and he will get an additional 7 million Euro. The additional amounts are added to the income of the last year of the old contract.

Start with the policy to stay with every team until the contract expires. To take into account the decreasing value of money over time we use a discount factor, which is chosen to be  $\alpha = 0.5$ . (This might seem unrealistic but keeps the calculations reasonable)

Use the policy improvement algorithm with discounting to *improve on* the starting policy, if possible.

- 5. As we know, Frasse is working on both large and small scale. As a *pro bono* he is now helping one of the street vendors for football shirts to design the optimal inventory level.
  - (a) The vendor sells shirts at a rate of 20 per day. He buys the shirts for 5 Euro each, and sells them for 25 Euro.

His expenses for vending and holding inventory equals 0.4 Euro per shirt (in inventory) and per day. Each time he orders new shirts he has to pay 100 Euro for the delivery.

How often should he order shirts, and how many shirts should he then order, if he wants to minimize the costs (per time unit) and avoid shortage.  $\dots$  (6p)

- (c) Consider now the case in (a) again, where we assume that the vendor only sells Zlatan Ibrahimovic shirts which he buys for 5 Euro each. Assume that the shirts will loose all their value if it is announced that Zlatan is changing to another team. Assume that the probability that this occurs is p = 0.001 times the length of the time interval in which it would occur, *i.e.*, in the next 10 days the probability would be 10p = 0.01.

Good luck!