# Marginal Allocation for Model 3

Let $\boldsymbol{S} = (s_1, s_2, \dots, s_n)^T$ = # spares at the bases

$$EBO(s_0, \boldsymbol{S}) = \sum_{j=1}^{n} EBO_j(s_0, s_j)$$ = Average # grounded aircrafts

For fixed $s_0$, $EBO(s_0, \boldsymbol{S})$ is a seperable integer-convex decreasing function

$$C(s_0, \boldsymbol{S}) = cs_0 + \sum_{j=1}^{n} cs_j$$ = Total cost of spare units

(For fixed $s_0$,) $C(s_0, \boldsymbol{S})$ is a seperable integer-convex increasing function.

Algorithm: Fix $s_0$ and determine $EBO_0(s_0)$ using the recursive algorithm for Model 1.

Start with $\boldsymbol{S}_{s_0}^{(0)} = (0, 0, \dots, 0)^T$ (minimizing $C(s_0, \boldsymbol{S})$) and apply the Marginal Allocation algorithm for Model 2, where

$j$ now corresponds to base $j$ (instead of $LRU_j$)

$c_j = c$ for $j = 1, 2, \dots, n$ cost for a spare unit

$T_j = T_{db} + EBO(s_0) \cdot \frac{1}{\lambda_0}$ expected pipeline times.

continued...

Let $F(s_0, k) = EBO(s_0, \mathcal{S}_{s_0}^{(k)})$ = optimal EBO for $s_0$ spares at C.D. and k spares at the bases.

Note: $C(s_0, \mathcal{S}_{s_0}^{(k)}) = cs_0 + \sum_{j=1}^{n} c\, s_j^{(k)} = cs_0 + ck$

since one spare unit is added at each iteration

Determine, for $s_0 = 0$

| | | |
|---|---|---|
| $\mathcal{S}_0^{(0)}$ | $EBO(0, \mathcal{S}_0^{(0)}) = F(0,0)$ | $C(0, \mathcal{S}_0^{(0)}) = 0$ |
| $\mathcal{S}_0^{(1)}$ | $EBO(0, \mathcal{S}_0^{(1)}) = F(0,1)$ | $C(0, \mathcal{S}_0^{(1)}) = c$ |
| $\mathcal{S}_0^{(2)}$ | $EBO(0, \mathcal{S}_0^{(2)}) = F(0,2)$ | $C(0, \mathcal{S}_0^{(2)}) = 2c$ |
| $\vdots$ | $\vdots$ | $\vdots$ |

for $s_0 = 1$

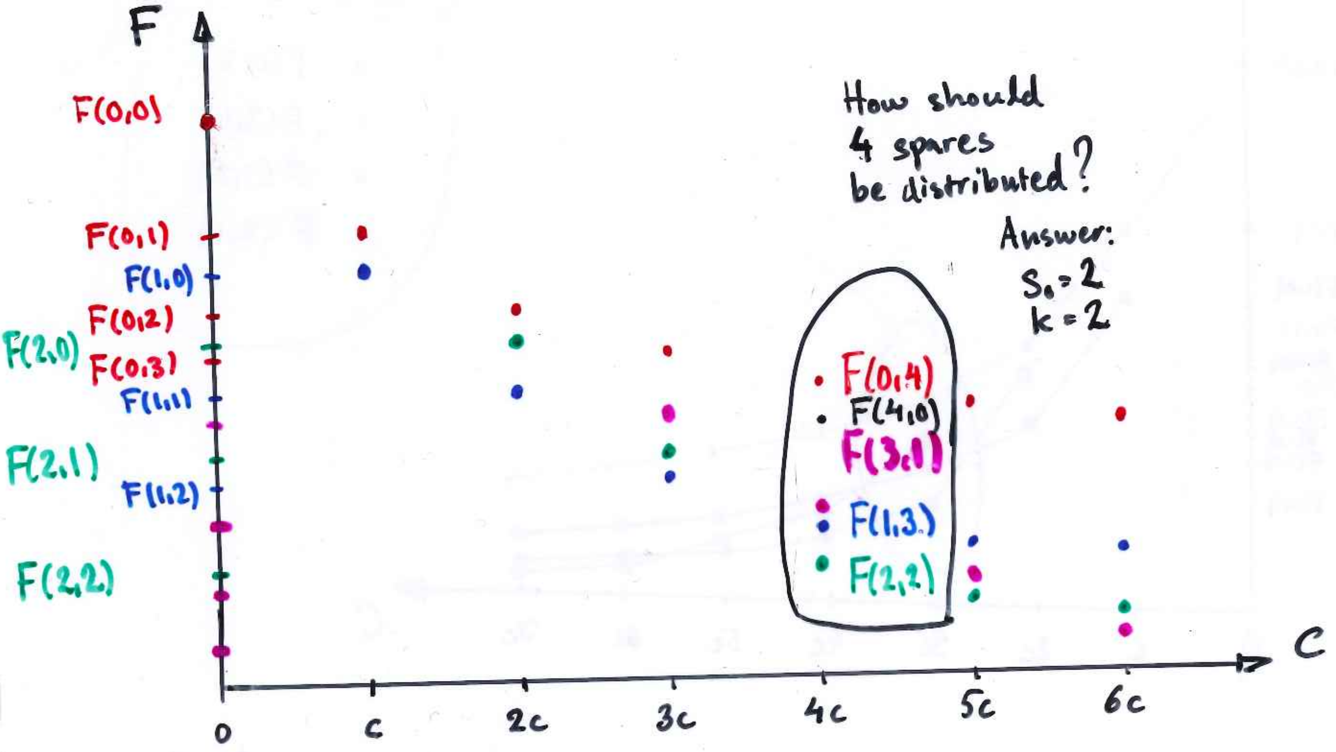
| | | |
|---|---|---|
| $\mathcal{S}_1^{(0)}$ | $EBO(1, \mathcal{S}_1^{(0)}) = F(1,0)$ | $C(1, \mathcal{S}_1^{(0)}) = c$ |
| $\mathcal{S}_1^{(1)}$ | $EBO(1, \mathcal{S}_1^{(1)}) = F(1,1)$ | $C(1, \mathcal{S}_1^{(1)}) = 2c$ |
| $\mathcal{S}_1^{(2)}$ | $EBO(1, \mathcal{S}_1^{(2)}) = F(1,1)$ | $C(1, \mathcal{S}_1^{(2)}) = 3c$ |
| $\vdots$ | $\vdots$ | $\vdots$ |

for $s_0 = 2$ ...

The last step is to compare $F(s_0, k)$ for different $s_0$ and $k$.

## What do we do when $S_0$ is allowed to vary?

We compare the solutions for all choices of $S_0$




For each value of C compare the efficient solutions for fixed $S_0$.

Let $F(\ell) = \min_{S_0} \{ F(S_0, \ell - S_0) \mid 0 \le S_0 \le \ell \}$ $\ell = 0, 1, \ldots$

Then $(0, F(0))$, $(c, F(1))$, $(2c, F(2))$, $\vdots$ are the best solutions for each choice of number of spare units

If the piecewise linear curve joining these points is convex it is the efficient curve for Model 3.

Otherwise, the efficient curve is obtained by generating the "southwestern" boundary of the convex hull of the points above.