KTH Matematik

5B1822: Geometric Systems Theory

Homework 2

Due November 29, 16:50pm, 2004

You may discuss the problems in group (maximal three students in a group), but each of you **must** write and submit your own report. Write the names of persons that you cooperated with.

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1. Consider the system

$$\begin{aligned}\dot{x} &= \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} u \\ y &= \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} x.\end{aligned}$$

(a) What is the zero dynamics? [1p]

(b) Use the Rosenbrock matrix to verify your computation of the transmission zero from (a). [2p]

(c) Solve the noninteracting control problem. [1p]

2. Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2x_1 - 3x_2 + w_1 \\ \dot{w}_1 &= 4w_1 - 5w_2 \\ \dot{w}_2 &= 5w_1 - 4w_2 \\ y &= x_1,\end{aligned}$$

(a) Compute the invariant subspace $x = \Pi w$ (the matlab command “lyap” can be used). [2p]

(b) What is $w_1(t)$ (which is the input to the system) for $w_1(0) = 0,\ w_2(0) = 1$? [1p]

(c) What is $y(t)$ *in stationarity* if $w_1(0) = 0,\ w_2(0) = 1$? [1p]

3. Consider the car steering example:

$$\begin{aligned}&\dot{\alpha}_f = -3\alpha_f + 0.4r \\ &\dot{\psi} = r \\ &\dot{r} = -0.8\alpha_f - 0.6\psi - r + d(t),\end{aligned}$$

where the driver's goal is to keep the orientation straight ($\delta_f = -k\psi$), $d(t)$ is a sinusoidal disturbance $a\sin(2t+\theta)$ with unknown amplitude and phase.

Design an output that is a linear combination of ψ and r, such that the output reconstructs the disturbance in stationarity. [2p]

4. Consider:

$$
\begin{aligned}
\dot{x}_1 &= \alpha x_1 - x_3 + w_1 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= u_2 \\
\dot{x}_4 &= x_2 - x_4 - 2u_1 + u_2 \\
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= -w_1 \\
\dot{w}_3 &= w_2 \\
e_1 &= x_2 - w_2 \\
e_2 &= x_4 - w_3
\end{aligned}
$$

(a) For $\alpha = 1$, find a control $u = Kx + Ew$ that solves the full information output regulation problem. (*Hint: check if the system is already in the normal form when setting* $w = 0$.) [2p]

(b) In the closed-loop system, for $x(0) = 0$ and $w_1(0) = 0,\ w_2(0) = 1,\ w_3(0) = 1$, plot x_2 vs x_4 for a while in Matlab until you see the pattern for the stationary response. What does it look like in stationarity? [1p]

(c) What are the real values of α such that the regulation problem is **not** solvable? [1p]