KTH Matematik

5B1822: Geometric Systems Theory

Homework 3

Due December 10, 16:50pm, 2004

You may discuss the problems in group (maximal three students in a group), but each of you **must** write and submit your own report. Write the names of persons that you cooperated with.

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1. Consider the system

$$\dot{x} = g_1 u_1 + g_2 u_2,$$

where

$$g_1 = \begin{pmatrix} \cos(x_3 + x_4) \\ \sin(x_3 + x_4) \\ \sin(x_4) \\ 0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

One can view this as a more complex vehicle steering system. Define:

$$Drive = g_1, \quad Steer = g_2, \quad Wriggle = [Steer, Drive], \quad Slide = \begin{pmatrix} -\sin(x_3) \\ \cos(x_3) \\ 0 \\ 0 \end{pmatrix},$$

where $[\cdot, \cdot]$ is the Lie Bracket.

- What is $[Steer, Wriggle]$ and $[Wriggle, Drive]$? [1p]
- Show that the system is locally strongly accessible and controllable. [1p]

2. Consider

$$\begin{aligned} \dot{x}_1 &= -x_1 + 2\mathbf{k}x_2 - x_2^3 \\ \dot{x}_2 &= \mathbf{k}x_1 - 2x_2 + x_1 x_2^3 \\ \dot{x}_3 &= x_2^3 - x_3. \end{aligned}$$

Decide the respective range of $\mathbf{k}$ such that

- The origin is exponentially stable, [1p]
- The origin is only (non-exponentially) asymptotically stable, [1p]
- The origin is unstable. [1p]

3. Consider in a neighborhood N of the origin

$$
\begin{aligned}
\dot{x}_1 &= -x_1^3 + x_3 \cos x_1 \\
\dot{x}_2 &= x_1 + w \\
\dot{x}_3 &= x_2 e^{x_1} + u \\
y &= x_1
\end{aligned}
$$

where w is disturbance.

- Is the DDP solvable? [1p]
- Is the system exactly linearizable (without considering the output) around the origin when w is set to zero? [1p]
- Is the system asymptotically stabilizable around the origin when w is set to zero? [1p]

4. Consider in a neighborhood N of the origin

$$
\begin{aligned}
\dot{x}_1 &= x_1^4 + 2u \\
\dot{x}_2 &= -x_2 + x_3^2 \\
\dot{x}_3 &= x_1^3 + x_3 + u \\
y &= x_3.
\end{aligned}
$$

- Convert the system locally into the normal form. [2p]
- Is the zero dynamics asymptotically stable? [1p]
- Can we use high gain output control to stabilize the system locally? [1p]