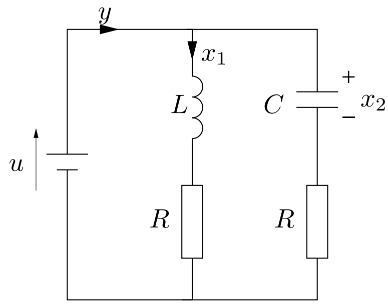
KTH Matematik

Homework 2
Mathematical Systems Theory, 5B1742
Spring 2006

1. Consider the electrical circuit in Figure 1



Figur 1: Kalman's example circuit.

(a) Determine a state space equation relating the input u (voltage) to the output y (current). Let x_1 (inductor current) and x_2 (capacitor voltage) be the states.

(b) Let $R = 1Ohm$ $L = 1H$ $C = 1F$. Determine a Kalman decomposition of the realization obtained in (a).

(c) Explain the result in (b).

2. Consider the transfer function

$$G(s) = \frac{1}{(s+1)(s+3)}\begin{bmatrix} 1 & 0 \\ -1 & 2(s+1)^2 \end{bmatrix}$$

(a) Determine the standard reachable realization.

(b) Determine the standard observable realization.

(c) Determine the McMillan degree.

(d) Use Ho's algorithm to determine a minimal realization.

Optional tasks:

(Oa) The standard reachable realization is not minimal. Determine the unobservable subspace.

(Ob) The standard observable realization is not minimal. Determine the reachable subspace.

3. For this exercise consider the discrete time system

$$x_{k+1} = Ax_k, \quad x_0 \text{ given}$$

and the discrete time Lyapunov equation

$$P = A^T PA + C^T C \tag{1}$$

Prove (or sketch the proof) of Theorem 4.2.4 in Lindquist and Sand. That is, prove that if (C, A) is observable then the following are equivalent

(i) A is stable, i.e. $|\lambda(A)| < 1$, for all eigenvalues of A.

(ii) $P > 0$

Hint: You may use that any discrete time Lyapunov equation

$$P = A^T PA + Q$$

has the solution $P = \sum_{k=0}^{\infty} (A^T)^k Q A^k$ when A is stable. It easy to see that if this P is bounded because when A is stable then there exists $c > 0$ and $0 < \rho < 1$ such that $\|A^k\| \leq c\rho^k$. Note that $P \geq 0$ if $Q \geq 0$. The main point is that the stronger conclusion that P is positive definite holds if $Q = C^T C$, where (A, C) is observable.