



KTH Matematik

## Computer Exercise 2

### Mathematical Systems Theory, 5B1742

### Spring 2006

Satisfactory completion of this computer exercise gives two (2) bonus credits for this year's final exam. The examination of the computer exercise is *oral and written*. Firstly the results should be presented in a written report (use a word processor) and secondly each student should be prepared to answer question about the exercise when it is handed back. Hand in the report at the latest on **May 12, 5:00pm**, and pick it up after agreement with Ulf Jönsson. Do attach your commented MATLAB code to your report! Cooperation in groups of no more than two students is allowed. Only one report for each group is required. The necessary MATLAB m-files can be found at:

[http://www.math.kth.se/optsys/studinfo/5B1742/5B1742\\_info.html](http://www.math.kth.se/optsys/studinfo/5B1742/5B1742_info.html)

The intention of this exercise is to exemplify how easy it is to apply the theory of the course using a computer. We will use the "Control System Toolbox" in MATLAB, which will save us lots of programming. Names written with bold font are command names in MATLAB.

Write **help control** to get a list of available functions in the "Control System Toolbox" or use the help browser.

## 1 Observers

In this exercise we will construct a full-order observer to the inverted pendulum system and use it to make an *output* feedback stabilization of the system.

1. Construct an observer with stable poles to the inverted pendulum system of Lab1.
2. Combine a stabilizing feedback with the observer above (as in the Figure on page 62 in the lecture notes ("Kurskompendiet") ). The combination is a new linear system with twice as many states ( $x$  and  $\hat{x}$ ), four additional outputs (the estimation errors, making the total 6)  $y = (x_1, x_3, x_1 - \hat{x}_1, x_2 - \hat{x}_2, x_3 - \hat{x}_3, x_4 - \hat{x}_4)$  and one input (the original one). Construct the new (bigger) system  $A_{new}, B_{new}, C_{new}, D_{new}$ . Is the new system minimal?
3. For the closed-loop system designed in (1) and (2), simulate a unit impulse response ( $v(0) = 1, v(t) = 0$  for  $t \neq 0$ ) with an initial condition

$$x_i(0) = 0, \quad \hat{x}_i(0) = 0.1, i = 1, 2, 3, 4,$$

using **lsim**. You use the following commands to construct the time axis and the unit impulse function

```
tvec = 0:.1:5;  
% Unit Impulse  
v = [1 zeros(1,length(tvec)-1)];  
x0 = 0.1*[ 0 0 0 0 1 1 1 1 ]';
```

Tune the locations of the closed-loop poles such that the following requirements are satisfied:

- (a) the input signal  $u = K\hat{x} + v$  has absolute value of less than 10 for all the time.
- (b) the settling time is about 10 time units or less.

In case you cannot achieve these requirements, present your “best” result. Plot (total 6) output signals and a input signal  $u$ .

Try to find some relationship between the choice of poles for observer and state feedback design and the resulting systems response. Explain this briefly in your report.

## 2 L-Q Optimal Control.

In this section we use the linearized pendulum equations from Lab 1.

Consider the following infinite time horizon LQ problem

$$\min \int_0^{\infty} x^T Q x + u^T R u \, dt$$

when

$$\dot{x} = Ax + Bu, \quad x(0) = x_0.$$

That is driving the system towards the equilibrium over infinite time while keeping controls and errors small.

1. Choose your own  $Q$  and  $R$  and calculate the optimal control (**are** or **care** might be useful).
2. Set the initial condition to be  $[0.1, \dots, 0.1]$ . Design an optimal controller such that the closed-loop system satisfies the following requirements.
  - The absolute value of the input signal  $u = Kx$  is at most 5 for all the time.
  - Settling time is around 10 time units or less.

Possibly, you need to tune  $Q$  and  $R$ . In case you cannot achieve these requirements, present your “best” result.

3. Solve the same problem as above with a more strict input condition (without settling time condition).
  - The absolute value of the input signal  $u = Kx$  is at most 1 for all the time.

If you cannot achieve this requirement, guess the reason why you cannot.

Try to find some relation between the tuning of  $Q$  and  $R$  and the resulting systems response. Explain this briefly in your report.

### 3 Kalman filter for parameter estimation

The Kalman filter is a widely used algorithm in control and signal processing. Applications are for example state estimation, sensor fusion and target tracking. Examples of more advanced target tracking are for example a camera tracking moving humans and radars tracking airplanes. The case we will study is tracking of a piecewise constant but unknown signal. Let  $x_k$  be a scalar that is a piecewise constant function of time where we do not know the values that  $x_k$  can assume and not the times when it changes. Let us say that  $|x_k| \leq 1$  though (this is for all lab groups to get comparable results). The measurements that we have are noisy measurements  $y_k$  of  $x_k$  defined by

$$y_k = x_k + dw_k \quad (1)$$

where  $w_k$  is  $N(0, 1)$  and  $d = 0.3$ . The task is to design a Kalman filter for estimating  $x_k$  given  $y_k$ . The following model puts the problem in a Kalman filter framework:

$$\begin{cases} x_{k+1} &= x_k + bv_k \\ y_k &= x_k + dw_k \end{cases} \quad (2)$$

where we have modeled  $x_k$  as a random walk. Now,  $b$  is a tuning parameter that you have to choose and it is part of the lab to study how this choice affects the filter performance. Remember that (2) is a *model* and that the true  $x_k$  and thus the measurements  $y_k$  must be generated off-line where you choose the piecewise constant  $x_k$  in some way you find suitable.

1. Implement a Kalman filter that estimates  $x_k$  (Theorem 9.1.10).
2. Feed the filter with observations as in (1), where  $x_k$  is piecewise constant (chosen by you) and  $w_k$  is random noise (use **randn**). Study the following aspects experimentally.
  - (a) How does  $P_0$  affect performance? (study the initial behavior).
  - (b) How does  $b$  affect performance?
  - (c) There seems to be a compromise between fast convergence to the true  $x_k$  and smooth estimates. Describe this and explain why.

*Good Luck! Don't hesitate to ask (by email or phone) if anything is unclear.*