KTH Matematik

5B1823: Geometric Control Theory

# Homework 1

Due November 15, 16:50, 2006

You may discuss the problems in group (maximal **two** students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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**1.** [3p]. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & -2 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} u \\ y &= (0\ 0\ 1\ 0)x, \end{aligned}$$

where $x = (x_1, x_2, x_3, x_4)^T$.

(a) Is the system controllable?

(b) Compute $\mathcal{V}^*$ and find all friends $F$ of $\mathcal{V}^*$.

**2.** [2p]. Consider the same system as in Problem 1. Suppose we are given a three dimensional space $XYZ$. We identify $x_1$ in the system as $x$, $x_2$ as $y$, $x_3$ as $z$ and $x_4$ as $\dot{z}$.

(a) What is the set of points on the $XY$ plane the origin can reach in finite time (by some control), via trajectories lying on the plane?

**3.** [5p]. Consider

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u + Ew, \\ y &= (a\ 1\ 0)x \end{aligned}$$

where $w$ is the disturbance.

(a) Derive the minimum constraint on $E$ such that $DDP$ is solvable for the cases $a = 2$ and $a = -2$. Find a state feedback $u = Fx + v$ that solves the $DDP$ problem for $a = -2$.

(b) Can we find a $u = Fx + v$ that solves the $DDP$ problem for any $E$ that meets the minimum constraint obtained above while makes the closed-loop system stable, i.e. $A + BF$ has only eigenvalues with negative real part (Discuss both the cases $a = 2$ and $a = -2$)?

(c) Can we find an output feedback $u = Ky + v$ that solves the respective DDP?

**4.** [5p]. Consider

$$\begin{aligned}
\dot{x}_1 &= -2x_1 + x_4 + u_1\\
\dot{x}_2 &= x_2 + 2u_2\\
\dot{x}_3 &= x_2 + x_4 + u_2\\
\dot{x}_4 &= x_3\\
y_1 &= x_1 - x_3\\
y_2 &= x_4
\end{aligned}$$

(a) What is the relative degree for the system?

(b) Convert the system into the normal form and compute the zero dynamics.

(c) What is the $\mathcal{R}^*$ contained in $Ker\ C$?