KTH Matematik

5B1823: Geometric Control Theory

Homework 3

Due December 8, 16:50pm, 2006

You may discuss the problems in group (maximal two students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. Consider the system

$$\dot{x} = g_1 u_1 + g_2 u_2,$$

where

$$g_1 = \begin{pmatrix} \cos(x_3 + x_4) \\ \sin(x_3 + x_4) \\ \sin(x_4) \\ 0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

One can view this as a more complex vehicle steering system. Define:

$$Drive = g_1, \quad Steer = g_2, \quad Wriggle = [Steer, Drive], \quad Slide = \begin{pmatrix} -\sin(x_3) \\ \cos(x_3) \\ 0 \\ 0 \end{pmatrix},$$

where $[\cdot, \cdot]$ is the Lie Bracket.

- What is $[Steer, Wriggle]$ and $[Wriggle, Drive]$? [1p]
- Show that the system is locally strongly accessible and controllable. [1p]

2. Consider

$$\begin{aligned} \dot{x}_1 &= \alpha x_1 + x_1 x_2^3 \\ \dot{x}_2 &= -x_1^2 + \beta x_2, \end{aligned}$$

where α, β are constants.

Determine the stability of the origin by dividing α and β into different ranges so that you use respectively

- the principle of stability in first approximation to analyze the stability; [1p]
- the center manifold theory to analyze the stability. [2p]

3. Consider in a neighborhood N of the origin

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_3 + x_1^2 + u \\ \dot{x}_2 &= -x_2 + 2x_3 + x_1 x_2 \\ \dot{x}_3 &= x_1^2 + x_2 + x_2^2 + u \\ y &= x_3. \end{aligned}$$

- Convert the system locally into the normal form. [2p]
- Can we use high gain output control to stabilize the system locally? [1p]
- Is the system exactly linearizable (without considering the output) around the origin? [2p]

4. Consider the headings of four birds:

$$\dot{\theta}_i = u_i, \ i = 1, 2, 3, 4.$$

Suppose each bird updates its heading angle using the consensus control discussed in Chapter 9. If the initial headings for bird 1, bird 2 and bird 3 are respectively $\theta_1(0) = \frac{\pi}{4}, \ \theta_2(0) = \frac{3}{8}\pi, \ \theta_3(0) = \frac{4{,}8}{8}\pi$,

- What should be the initial heading of bird 4 if the flock can eventually fly towards north ($\theta = \frac{\pi}{2}$)? [1p]
- Suppose each bird's viewing field is $(-\frac{\pi}{4}, \frac{\pi}{4})$. Design a consensus control for each bird (use matlab to verify that it works). [2p]