KTH Matematik

**Homework 2: 5B1872: Optimal Control Spring 2006**

**Grading: You may use $\dfrac{\text{your credit}}{20}$ extra points on the exam.**

**1.** Consider the time optimal control problem

$$\min t_f \quad \textit{subj.to} \quad \begin{cases} \ddot{y} = u, \ |u| \leq 1, \ t_f \geq 0 \\ y(0) = \alpha, \ y(t_f) = 0 \\ \dot{y}(0) = \beta, \ \dot{y}(t_f) \geq 0 \end{cases}$$

An interpretation in terms of the rocket car example is that we want to pass the origin with positive velocity as soon as possible.

($a$) What are the possible optimal switching sequences?

($b$) Sketch the optimal solution in a phase plane plot where the switching curve and the control values should be clearly indicated.

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10p

**2.** A lifeguard is standing on the beach at position $(0, 0)$ when he discovers a person in distress at position $(a, b)$. The equations of motion of the lifeguard are

$$\begin{aligned} \dot{x}(t) &= v(y(t)) \cos(u(t)), \quad x(0) = 0, \quad x(T) = a > 0 \\ \dot{y}(t) &= v(y(t)) \sin(u(t)), \quad y(0) = 0, \quad y(T) = b > 0 \end{aligned}$$

where his speed depends on the distance to the beach

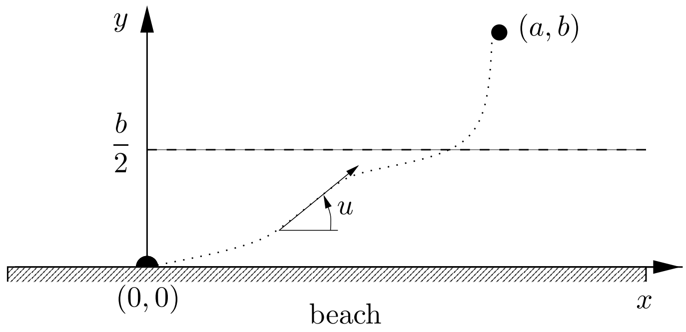
$$v(y) = \begin{cases} 1, & 0 \leq y \leq \dfrac{b}{2} \\ \dfrac{1}{2}, & \dfrac{b}{2} < y \leq b \end{cases}$$

Find the optimal control function $u(t)$ such that the lifeguard reaches the person in shortest time. Do the numerical calculation of the intersection point with the line $y = \frac{b}{2}$ for the case when $b = 4$ and $a = 2$. Note that the function $v(y)$ is not smooth, therefore you have to treat the two regions separately and then combine using, for example, dynamic programming.

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (10p)

**3.** Determine the bang-bang control for the following time optimal control problem

$$\min T \quad \text{subj. to} \quad \begin{cases} \dot{x} = x^2 - \dfrac{1}{4} - xu \\ x(0) = \dfrac{1}{2}, \ x(T) = -\dfrac{1}{2} \\ |u| \leq 1 \end{cases}$$




Figur 1: The lifeguard is positioned at $(0,0)$ and wants to reach the person who is fixed at $(a,b)$ in as short time as possible.

You are allowed to compute the switching time numerically.
*Hint: Study the switching function. Use that the solution of a differential equation* $\dot{\lambda}(t) = g(t)\lambda(t)$ *has constant sign.*

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (10p)