KTH Matematik

Homework 1: 5B1872: Optimal Control Spring 2006

Grading: You may use $\frac{\text{your credit}}{20}$ extra points on the exam.

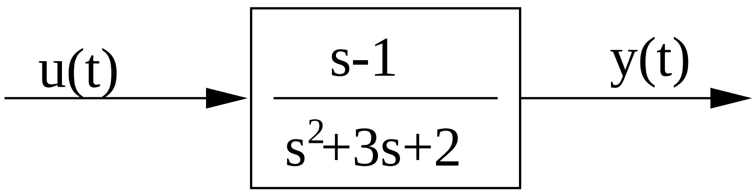
1. Consider the system in Figure 1.

(a) Determine a diagonal state space realization of the linear system (the A matrix should be diagonal)

$$\dot{x} = Ax + Bu, \ x(0) = 0$$
$$y = Cx$$

Hint: Do a partial fraction expension of $G(s) = \frac{s-1}{s^2+3s+2}$

. . . . (3p)



Figur 1: A signal is sent through a filter with transfer function $G(s) = \frac{s-1}{s^2+3s+2}$.

(b) Assume the control is constrained as $|u(t)| \leq 1$. Determine an explicit expression for the optimal input (as a function of t_f) such that the output $y(t_f)$ is maximized, i.e. solve

$$\max Cx(t_f) \quad \text{subject to} \quad \begin{cases} \dot{x} = Ax + Bu, \ x(0) = 0 \\ |u(t)| \leq 1 \end{cases}$$

. . . . (7p)

2. Consider the DC-servo in Figure 2. A mathematical model for the relation between the voltage, u, and the angle of the shaft, θ, is

$$\ddot{\theta} + \dot{\theta} = u$$

We assume that the servo starts at rest, i.e. $\theta = \dot{\theta} = 0$. We want to design a voltage signal such that the angle θ tracks a sinusoid over the time interval $[0, 8\pi]$. One way to do this is to solve the following optimization problem

$$\min q_0[\theta(8\pi)^2 + \dot{\theta}(8\pi)^2] + \int_0^{8\pi} [q(\theta(t) - \sin(t))^2 + ru(t)^2]dt$$

subject to the linear dynamics

$$\ddot{\theta}(t) + \dot{\theta}(t) = u(t), \quad \theta(0) = \dot{\theta}(0) = 0 \tag{1}$$

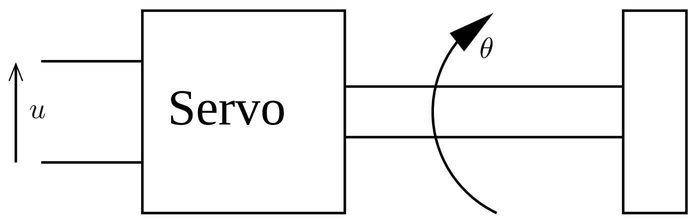
The terminal cost is used in order to bring shaft close to rest at $\theta = 0$ at the terminal time.

a Use Matlab to solve the problem. Choose q, q_0 and r such that $|u(t)| \leq 2$ and such that good tracking is obtained. Plot your solutions $\theta(t)$ and $u(t)$. Include the Matlab code in the solution you hand in.

. . . . (7p)

b How good solution can you get if q is fixed to be $q = 1$?

. . . . (3p)



Figur 2: Control the DC-servo such that the angle θ tracks a sinusoidal.

3. A decision maker must choose between two activities over a time interval $[0, t_f]$. Each activity earns a reward at rate $g_k(t)$, $k = 1, 2$. Every switch between the two activities costs $c > 0$. As an example, the reward for starting with activity 1, switch to activity 2 at time t_1 and back to 1 at time $t_2 > t_1$ earns the total reward

$$\int_0^{t_1} g_1(t)dt + \int_{t_1}^{t_2} g_2(t)dt + \int_{t_2}^{t_f} g_1(t)dt - 2c$$

We want to find a switching sequence that maximize the total reward. Switching can only occur inside the the interval $(0, t_f)$.

Assume the function $g_1(t) - g_2(t)$ changes sign a finite number of times in the interval $(0, t_f)$.

(*a*.) Formulate the problem as a sequential optimization problem and then formulate the corresponding DP recursion.

. . . . (6p)

(*b*.) Solve the dynamic programming problem in (*a*) for the case when $c = 2$, $t_f = 3$ and

$$g_1(t) = \begin{cases} 4, & 0 \leq t \leq 1 \\ 0, & 1 \leq t \leq 2 \\ 5, & 2 \leq t \leq 3 \end{cases}, \qquad g_2(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 6, & 1 \leq t \leq 2 \\ 2, & 2 \leq t \leq 3 \end{cases}$$

. . . . (4p)