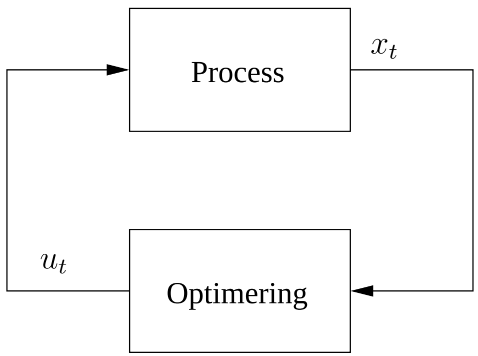
Model Predictive Control



The Algorithm

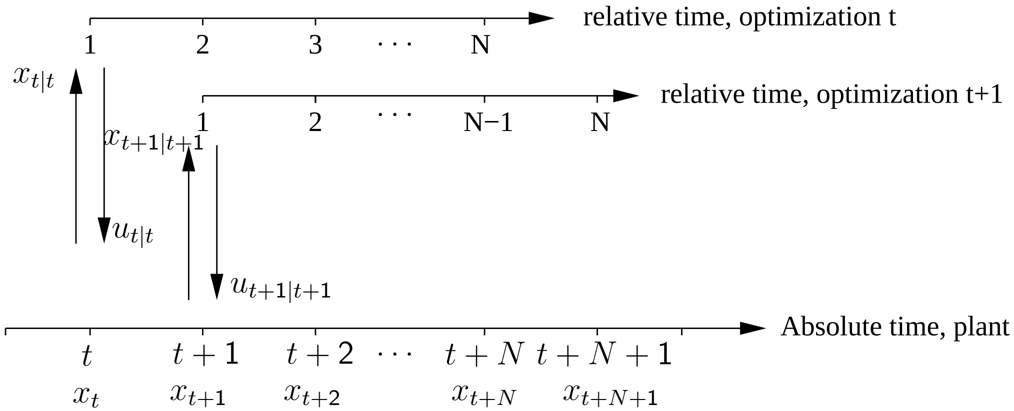
1. Measure $x_{t|t} := x_t$.
2. Determine $u_{t|t}$ by solving

$$\min \sum_{k=0}^{N-1} f_0(x_{t+k|t}, u_{t+k|t}) \quad \text{subj. to} \quad \begin{cases} x_{t+k+1|t} = f(x_{t+k|t}, u_{t+k|t}), \\ x_{t+k|t} \in \mathbf{X}_k, \; u_{t+k|t} \in \mathbf{U}_k \end{cases}$$

3. Apply $u_t := u^*_{t|t}$
4. Let $t := t + 1$ and go to 1.

$x_{t+k+1|t} = f(x_{t+k|t}, u_{t+k|t})$ is the predicted state given $x_{t|t}$.

Absolute and Relative Time Axis



Properies of MPC

(+) Feedback solution

(+) Can handle state constraints and control constraints

(+) May consider mixed continuous/discrete variables (Hybrid control)

(−) Stability and feasibility cannot be guaranteed in general. However, MPC gives closed loop stability under certain reasonable assumptions.

(−) Computional complexity may be severe in general. However, MPC is tractable in many applications.

- Power systems and power electronics
- Process control
- Active suspension

Online Optimization Versus Explicit MPC

1. Online optimization: Solve on-line in run-time the optimization $\min_{U_t} J(x_{t|t}, U_t)$, where

$$U_t = (u_{t|t}, u_{t+1|t}, \ldots, u_{t+N-1|t})$$

$$J(x_{t|t}, U_t) = \min \sum_{k=0}^{N-1} f_0(x_{t+k|t}, u_{t+k|t})$$

$$\text{subj. to} \quad \begin{cases} x_{t+k+1|t} = f(x_{t+k|t}, u_{t+k|t}), \\ x_{t+k|t} \in \mathbf{X}_k, \ u_{t+k|t} \in \mathbf{U}_k \end{cases}$$

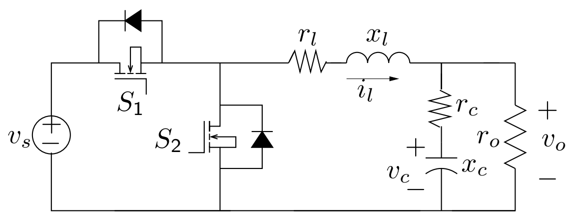
2. Find explicit solution

$$U_t^* = (\mu(1, x_{t|t}), \ldots, \mu(N-1, x_{t+N-1|t})) = \text{argmin}_{U_t} J(x_{t|t}, U_t)$$

and use $u_{t|t}^* = \mu(x_{t|t}) := \mu(1, x_{t|t})$ as state feedback function.

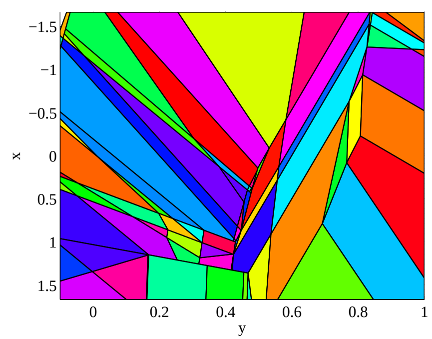
- The explicit solution is possible to compute in case of
  1. Linear cost, linear dynamics, and linear constraints.
  2. Quadratic cost, linear dynamics, and linear constraints (harder)
- The optimal solution is a piecewise linear map or look-up table.

Step-up Converter



$$\dot{x}(t) = \begin{cases} Fx(t) + fv_s, & kT_s \le t < (k + d[k])T_s, \\ Fx(t), & (k + d[k])T_s \le t < (k+1)T_s. \end{cases}$$

$$F = \begin{bmatrix} -\frac{1}{x_\ell}(r_\ell + \frac{r_o r_c}{r_o + r_c}) & -\frac{1}{x_\ell}\frac{r_o}{r_o + r_c} \\ \frac{1}{x_c}\frac{r_o}{r_o + r_c} & -\frac{1}{x_c}\frac{1}{r_o + r_c} \end{bmatrix}, \quad f = \begin{bmatrix} \frac{1}{x_\ell} \\ 0 \end{bmatrix}.$$



- Feedback solution defined over 633 polyhedral regions in state space
- For details, see T. Geyer, G. Papafotiou, and M. Morari, "On the optimal control of switch-mode dc-dc converters," in *Hybrid Systems: Computation and Control*, ser. LNCS, R. Alur and G. Pappas, Eds. Springer, 2004, vol. 2993, pp. 342–356.